(2 points) Suppose there are $n = 4$ advertisers with $L = [1, 5, 8, 2]$ and $P = [10, 20, 40, 5]$. Suppose the total time available is $W = 5$ minutes.

What advertisements and in what fractions must you show to maximize the revenue earned?

Provide your answer in the form of an array $X[1..n]$, where each element represents the time (in minutes) of the advertisement from the i-th advertiser. For example, $[0, 0, 0, 2]$ would indicate that you would only play the fourth advertiser's video for two minutes.

$X[1] =$ 1 ← 10
$X[2] =$ 0
$X[3] =$ 4 ← 20
$X[4] =$ 0

30

$\frac{P}{L} = [10, 4, 5, 2.5]$

3.

What follows is a proof that the greedy strategy of sorting the advertisers in decreasing order with respect to the amount they are willing to pay per minute ($P[i]/L[i]$) and distributing the time slot in this order is optimal.

Proof: Let $X_o[1..n]$ be an optimal solution to the problem that is different from the greedy solution.

Let $X_g[1..n]$ be a greedy solution to the problem.

Let i and j be two distinct advertisers such that $X_o[i] \geq X_o[j]$, but $X_g[i] \leq X_g[j]$.

~~Suppose we swapped the screen times of j and i in O.~~

The original revenue generated by i and j was $X_o[i]\frac{P[i]}{L[i]} + X_o[j]\frac{P[j]}{L[j]}$.

After swapping, the revenue generated by i and j becomes $X_o[j]\frac{P[i]}{L[i]} + X_o[i]\frac{P[j]}{L[j]}$.

The difference in revenue is then $(X_o[j] - X_o[i])\frac{P[i]}{L[i]} - (X_o[j] - X_o[i])\frac{P[j]}{L[j]}$.

Since $X_g[i] \leq X_g[j]$, we have $\frac{P[i]}{L[i]} \leq \frac{P[j]}{L[j]}$. So, the difference in revenue is non-negative.

The greedy solution is thus shown to be as good as the optimal solution, so by the exchange argument, the greedy solution must also be optimal.

□

3. (4 points) Suppose you have the input $n = 5$, $W = 10$, $L = [7, 2, 4, 5, 1]$, and $P = [10, 2, 2, 4, 1]$, and An optimal solution to this problem is $X_o = [0, 0, 4, 5, 1]$. Suppose the greedy algorithm chose $X_g = [0, 1, 4, 5, 0]$.

What are i and j such that $X_o[i] \geq X_o[j]$, but $X_g[i] \leq X_g[j]$?
(notice that the arrays are ~~indexed starting at 1)~~

$X_o[1] = 0$
$X_o[2] = 0$

$i \in \{1, 2, 3, 4, 5\}$ $i = 1$ $X_g[1] = 1$
$j \in \{1, 2, 3, 4, 5\}$ $j = 2$ $X_g[2]$

What are the revenue generated by i and j in the optimal solution before swapping and after swapping?

Revenue before swapping = $X_o[1]\frac{P[1]}{L[1]} + X_o[2]\frac{P[2]}{L[2]} =$
Revenue after swapping =

2. (4 points) A possible approach to solving this problem is to use a greedy strategy. we want to maximize our revenue, it may be reasonable to prioritize advertisers willing to pay the most amount. As a result, consider the following strategy:

"Sort the advertisers in decreasing order with respect to the amount they are willin Distribute the time slots in this order."

For example, if $n = 3$, $W = 5$, $L = [1, 2, 5]$, and $P = [5, 10, 40]$, our strateg indicate that we should allocate the entire amount of time to the third advertiser, r a revenue of 40, which is the maximum amount for this set of advertisers. Can you a counterexample where this strategy breaks down?

Provide your answer in the form of the problem parameters n, W, L, and P. Also an array X_W, where each element represents the time (in minutes) allocated advertisement from the i-th advertiser with the strategy described in this problem array X_C which represents the optimal solution.

$n =$
$W =$
$L =$
$P =$

$X_W =$
$X_C =$

2

w/ groups:
verify answers
check that given X_o, X_g are optimal in 3

with table

① If you had to take the whole ad (instead of fractions), would price-per-time greedy still be optimal? no!

$W = 10$ $P = [7, 3, 5]$ $\frac{P}{L} = [1, \frac{3}{4}, \frac{5}{6}]$

$L = [7, 4, 6]$

greedy: $[7, 0, 0]$

② Does every computational problem have a correct greedy algorithm?