P and NP are both sets of decision problems.

$P \subseteq NP$

(a) (2 points) Define the set P.

The set of all decision problems that can be solved in polynomial time

there is an alg.

(b) (3 points) Define the set NP.

The set of all decision probs that can be verified in polynomial time.

The next questions will be about the following decision problems:

- k-Independent Set: Given a graph $G = (V, E)$, does G contain a size k independent set $X \subseteq V$ with no pair of vertices $u, v \in X$ connected by an edge $(u, v) \in E$?
- Connectivity: Given a graph $G = (V, E)$ and nodes $s, t \in V$, is s connected to t, that is, is there a path between s and t?
- Array membership: Given an array $A[1..n]$ of integers and an integer x, is x in A?

1 point each:

on PL

(a) Is the k-Independent Set problem in NP? Circle one: yes or no

(b) Is the Connectivity problem in NP? Circle one: yes or no

(c) Is the Connectivity problem in P? Circle one: yes or no

(d) Is the array membership problem in P? Circle one: yes or no

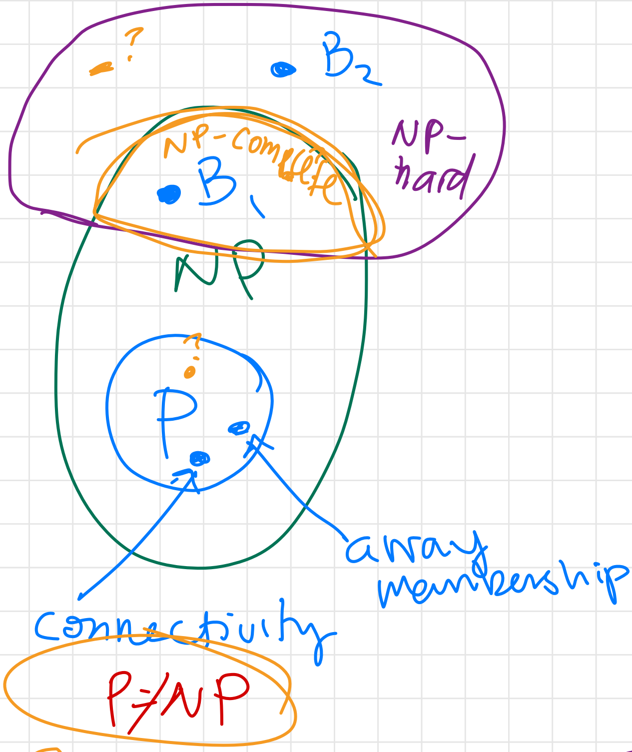
(e) Is the array membership problem in NP? Circle one: yes or no

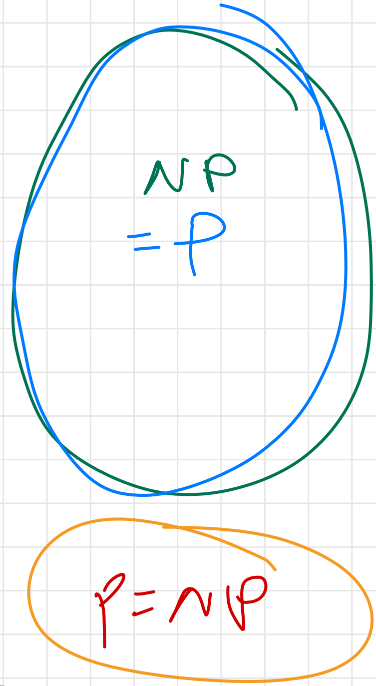
Verification

- define a certificate
- give a poly time alg to check that the certificate is valid

eg. a proposed IS
path
VC

$P \subseteq NP$ ... but is $P = NP$ or $P \neq NP$?





A problem B is NP-hard if for all problems A in NP, $A \leq_p B$.
If you could solve B fast, you could solve A fast.

A problem is NP-complete if:

① it is in NP

② it is NP-hard

* To show a problem B is NP-hard, show that an NP-hard problem C reduces to it. $C \leq_p B$.

Boolean Satisfiability Problems (SAT)

SAT: boolean formula.

is there an assignment to the boolean vars that evaluates to T?

$(x_1 \wedge x_2) \vee (x_1 \wedge \overline{x_3}) = T$?

"and" "or"

F F

$x_1 = T$ or F

$x_2 = T$ or F

$x_3 = T$ or F

$T \wedge T = T$
$T \wedge F = F$
$F \wedge T = F$
$F \wedge F = F$

$T \vee T = T$
$T \vee F = T$
$F \vee T = T$
$F \vee F = F$

is there a satisfying assignment?

can you come up w/ an input to SAT that is not satisfiable?

$\vee, \wedge, \overline{x}$

not x

F T F
T F F

$x_1 \wedge \overline{x_1}$

$x_1 = T$
$x_1 = F$

is SAT $\in$ NP ? ① yes ② no