before: giving algorithms

now: show that there (probably) aren't polynomial algorithms for problems

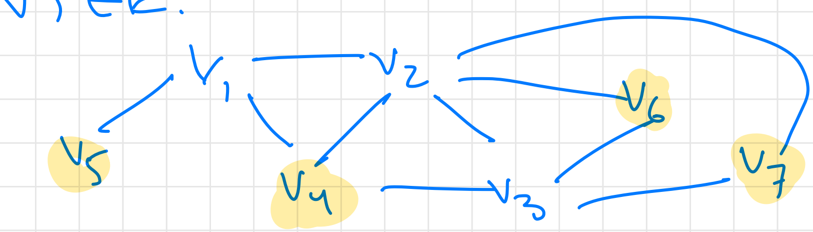
reductions: $A \le_p B$ "A reduces to B"

B is at least as hard as A

Example problems

① Independent Set, IS

Given an undirected graph $G = (V, E)$, $X \subseteq V$ is an IS if no pair of nodes $u, v \in X$ is connected by an edge $(u,v) \in E$.

$G =$ 

is $X = \{v_5, v_4\}$ an IS? yes.

is $(v_5, v_4) \in E$

$\emptyset \subseteq V$

- Is $\emptyset = \{\} =$ the empty set an IS? yes
- What is the largest IS?

opt prob: give size of largest IS in G. 4

search prob: give a set $X \subseteq V$ that is as large as possible

$\{v_4, v_5, v_6, v_7\}$

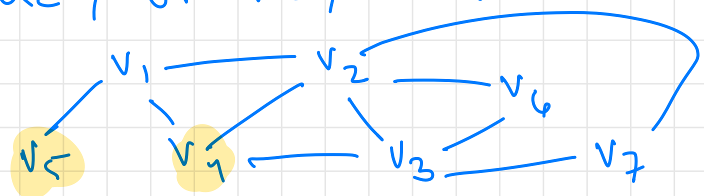
decision prob: is there a size k IS for G?

is there a size 5 IS in G? no.

k-IS.

② Vertex Cover, VC

Given an undirected graph $G=(V,E)$, $Y \subseteq V$ is a VC if every edge $(u,v) \in E$ has $u \in Y$ or $v \in Y$.



is there a size 4 IS in G?

is $\emptyset$ a VC for G?

is $\{v_5, v_4\}$ a VC for G?

is V a VC for G?

decision prob: is there a size k VC for G?

Let's do a reduction!

k-IS $\leq_p$ k-VC "k-IS reduces to k-VC"

Let $G = (V, E)$ and k be an input to k-IS.

Let $G' = G$ and $k' = |V| - k$. ← e.g., $7 - 4 = 3$.

Pass G' and k' into solver for k-VC.

Return answer from k-VC solver as answer to k-IS.

Why does this work? Vertex covers and Ind. sets are complements.

Is the reduction poly time?

- make G' linear
- make k' constant

P is the set of all decision problems that can be solved in polynomial time.

To prove a problem is in P, give a polynomial time alg for it.

What problems do we already know that are in P?

max points on GL
stable matching
multiplication

... every problem so far!

stable matching $\in$ P

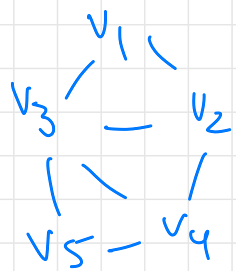
NP is the set of decision problems that can be verified in polynomial time.

How to prove that a problem is in NP?

Show how to give certificate and a verifier for the problem.

ex Show that k-VC $\in$ NP.

$G, k=2$



$S = \{v_1, v_2\}$

Certificate: S, a proposed VC for G.

verifier: alg to check that S is a k-VC for G.

Need to check:

- $|S| = k$
- $S \subseteq V$
- $\forall (u,v) \in E$: $u \in S$ or $v \in S$

If $A \in P$, $A \in NP$?

Quiz:

definition of P NP

Given a prob, is it in P? in NP?