

Name \_\_\_\_\_

## CSCI 332, Fall 2025

### Quiz 5

Fill in the following to prove that all trees have one more node than they have edges.

*Proof*

(1 point) Universal declaration: Let  $T$  be an arbitrary \_\_\_\_\_.

(2 points) Inductive hypothesis (for this proof, use “fewer nodes than” for your definition of “smaller than”):

There are two cases:

(2 points) Base case: Suppose  $T$  has one node. (you fill in the rest up to wrap-up sentence)

So  $T$  has one more node than it has edges.

(5 points) Inductive case: Suppose  $T$  has more than one node. (you fill in the rest. Points divided as follows: 1 point each for create a smaller tree  $T'$ , applying the inductive hypothesis to  $T'$ , explaining how to get from  $T'$  back to  $T$ , and 2 points for explaining how the number of nodes and edges change when going from  $T'$  to  $T$ .)

So  $T$  has one more node than it has edges.