Name ____

CSCI 332, Fall 2025
Quiz 6

You are running a TV station, and need to decide which advertisements to show during an open timeslot of W minutes during a prime-time show.

There are n advertisers, and each advertiser i has their video advertisement and how much they are willing to pay for their ad to be shown. You are given an array $L[1..n]$ and an array $P[1..n]$, where each element $L[i]$ and $P[i]$ are the video length and amount for each advertiser i, respectively. Since this is a prime-time slot, the advertisers are also willng to accept a partial showing of their ad—However, if only an x fraction of their advertisement is shown, they are only willing to pay $x \cdot P[i]$ dollars.

1. (2 points) Suppose there are $n = 4$ advertisers with $L = [1, 5, 8, 2]$ and $P = [10, 20, 40, 5]$. Suppose the total time available is $W = 5$ minutes.

   What advertisements and in what fractions must you show to maximize the revenue earned?

   Provide your answer in the form of an array $X[1..n]$, where each element represents the time (in minutes) of the advertisement from the i-th advertiser. For example, $[0, 0, 0, 2]$ would indicate that you would only play the fourth advertiser's video for two minutes.

   $X[1] =$
   $X[2] =$
   $X[3] =$
   $X[4] =$

2. (4 points) A possible approach to solving this problem is to use a greedy strategy. Because we want to maximize our revenue, it may be reasonable to prioritize advertisers that are willing to pay the most amount. As a result, consider the following strategy:

   "Sort the advertisers in decreasing order with respect to the amount they are willing to pay. Distribute the time slots in this order."

   For example, if $n = 3$, $W = 5$, $L = [1, 2, 5]$, and $P = [5, 10, 40]$, our strategy would indicate that we should allocate the entire amount of time to the third advertiser, receiving a revenue of 40, which is the maximum amount for this set of advertisers. Can you think of a counterexample where this strategy breaks down?

   Provide your answer in the form of the problem parameters n, W, L, and P. Also provide an array X_W, where each element represents the time (in minutes) allocated to the advertisement from the i-th advertiser with the strategy described in this problem, and an array X_C which represents the optimal solution.

   $n =$
   $W =$
   $L =$
   $P =$

   $X_W =$
   $X_C =$

What follows is a proof that the greedy strategy of sorting the advertisers in decreasing order with respect to the amount they are willing to pay per minute ($P[i]/L[i]$) and distributing the time slot in this order is optimal.

Proof: Let $X_o[1..n]$ be an optimal solution to the problem that is different from the greedy solution.

Let $X_g[1..n]$ be a greedy solution to the problem.

Let i and j be two distinct advertisers such that $X_o[i] \geq X_o[j]$, but $X_g[i] \leq X_g[j]$.

Suppose we swapped the screen times of j and i in O.

The original revenue generated by i and j was $X_o[i]\frac{P[i]}{L[i]} + X_o[j]\frac{P[j]}{L[j]}$.

After swapping, the revenue generated by i and j becomes $X_o[j]\frac{P[i]}{L[i]} + X_o[i]\frac{P[j]}{L[j]}$.

The difference in revenue is then $(X_o[j] - X_o[i])\frac{P[i]}{L[i]} - (X_o[j] - X_o[i])\frac{P[j]}{L[j]}$.

Since $X_g[i] \leq X_g[j]$, we have $\frac{P[i]}{L[i]} \leq \frac{P[j]}{L[j]}$. So, the difference in revenue is non-negative.

The greedy solution is thus shown to be as good as the optimal solution, so by the exchange argument, the greedy solution must also be optimal.

□

3. (4 points) Suppose you have the input $n = 5$, $W = 10$, $L = [7, 5, 45, 6]$, and $P = [10, 5, 2, 4, 6]$, An optimal solution to this problem is $X_o = [0, 2, 0, 4, 4]$. Suppose the greedy algorithm chose $X_g = [0, 5, 4, 0, 1]$.

   What are i and j such that $X_o[i] \geq X_o[j]$, but $X_g[i] \leq X_g[j]$?
   (notice that the arrays are indexed starting at 1)

   $i =$
   $j =$

   What are the revenue generated by i and j in the optimal solution before swapping and after swapping?

   Revenue before swapping =
   Revenue after swapping =