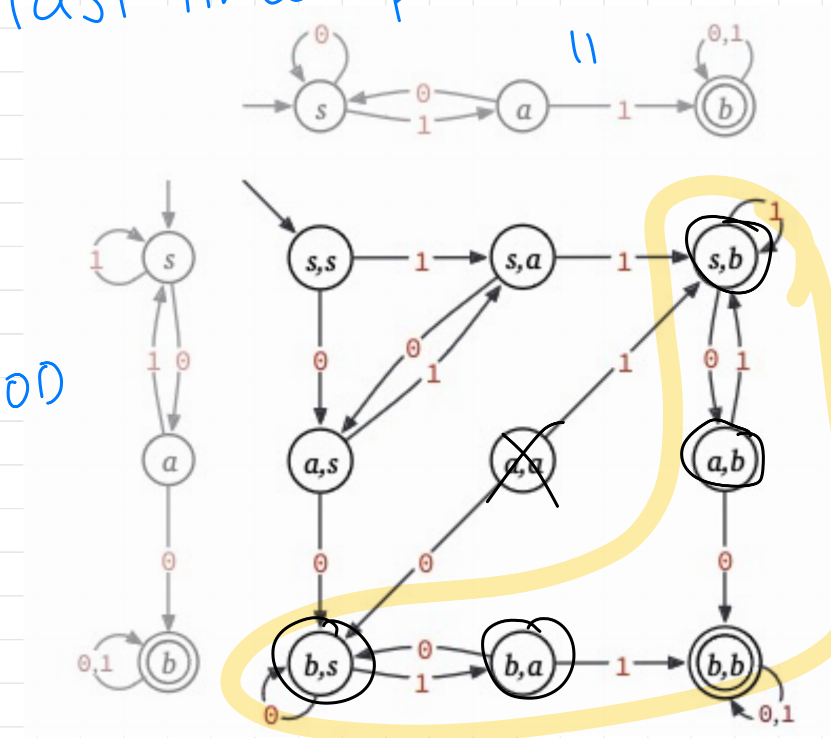
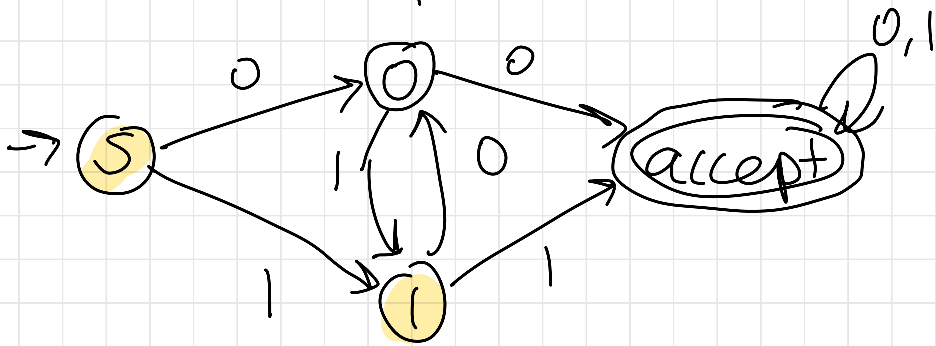
last time: product construction



00 or 11 = L



Is this the smallest DFA for L

Two states q, q' are <u>distinguishable</u> iff there is a string w s.t. $\delta^*(q, w)$ or $\delta^*(q', w)$ is accepting but not both.

s and accept are distinguished by 1
s and 0 are " " 0
s and 1 " " " 1
0 and 1 " " " 0 or 1
0 and accept " " " 1
1 and accept " " " ε

Two strings x and y are distinguishable with respect to language L iff there exists string z s.t. $xz \in L$ or $yz \in L$ but not both.

$L = (0+1)^*(00+11)(0+1)^*$

0 and 1 distinguishable?

$\downarrow$ $\downarrow$

x y

idea: $z = \varepsilon$

$xz = 0\varepsilon = 0 \notin L$

$yz = 1\varepsilon = 1 \notin L$

idea: $z = 1$

$xz = 01 \notin L$

$yz = 11 \in L$

A <u>fooling set</u> for language L is a set of strings that are all mutually distinguishable.

$F = \{\varepsilon, 0, 1, 00\}$ is a fooling set for $L = (0+1)^*(00+11)(0+1)^*$.

$\varepsilon, 0$	distinguished by	0
$\varepsilon, 1$		1
$\varepsilon, 00$		ε
$0, 1$ ✓		
$0, 00$		ε
$1, 00$		ε

For any language,

min # of states in a DFA accepting that language = max # of strings in a fooling set for that language

what if there is an infinite fooling set?

then any DFA accepting that language has infinite states...

so it can't exist!

Kleene's Theorem:

0^*1^* regular = automatic

#

w^n means $\underbrace{w \cdot w \cdot w \cdots \cdot w}_{n \text{ times}}$

$L = \{0^n 1^n : n \geq 0\}$ $\varepsilon, 01, 0011,$

let $F = \{\varepsilon, 0, 00, 000, 0000$ $0 \notin L$

F is a fooling set for L. $011 \notin L$

$\varepsilon, 0$ distinguished by 1 $1 \notin L, 01 \in L$

$\varepsilon, 00$ 11 $11 \notin L$ $0011 \in L$

$0, 00$ $\quad$ $11 \in L$

$\varepsilon, 000$

$0, 000$ $\quad$ 111

$00, 000$

$\vdots$

let $F = 0^* = \{\varepsilon, 0, 00, 000, 0000, \ldots\}$

$0^i, 0^j$ for $i \geq 0$, $j \geq 0$, $i \neq j$

let $z = 1^i$

$0^i 1^i \in L$

$0^j 1^i \notin L$ $\quad$ because $j \neq i$

F is infinite, so L is not regular!

Theorem: $L = \{ 0^n 1^n : n \geq 0 \}$ is not regular.

Proof:

Let $\boxed{F = 0^*}$

Let x, y be arbitrary different elements of F.

So $\boxed{x = 0^i}$ and $\boxed{y = 0^j}$ for $i \geq 0$, $j \geq 0$, $i \neq j$.

Let $\boxed{z = 1^i}$.

- $xz = \boxed{0^i 1^i} \in L$
- $yz = \boxed{0^j 1^i} \notin L$ because $i \neq j$

So z distinguishes x and y.

So F is a fooling set for L.

But F is infinite, so L is not regular.

Consider L = palindromes = $\{w : w = rev(w)\}$

001010100

Theorem: L is not regular.

Proof. Consider the set $F = \{0^n 1 : n \geq 0\}$.

Let x, y be arbitrary strings from F with $x \neq y$.

Then $x = 0^i 1$ and $y = 0^j 1$ for $i \neq j$.

Let $z = 0^i$.

- then $xz = 0^i 1 0^i \in L$
- but $yz = 0^j 1 0^i \notin L$

So z is a distinguishing suffix for x, y.

So F is a fooling set for L.

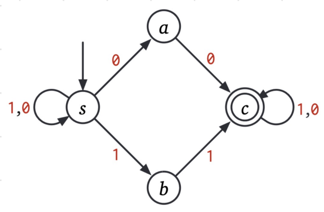
Because F is infinite, L is not regular.

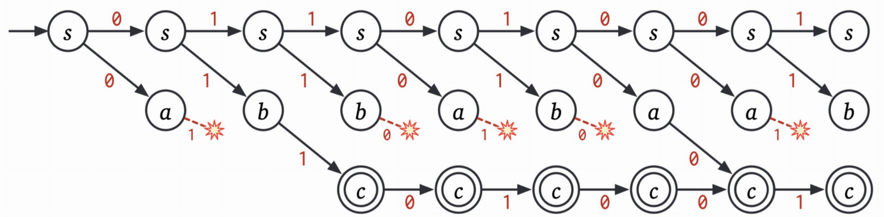
could I have used

$F = \{0^n : n \geq 0\} = 0^*$

$x = 0^i$, $y = 0^j$ $\quad z = 10^i$

$L = 0^* = \{0^n : n \geq 0\}$ regular or no?





Running our example NFA on the input string `01101001`.