Where we've been

- what is computable?
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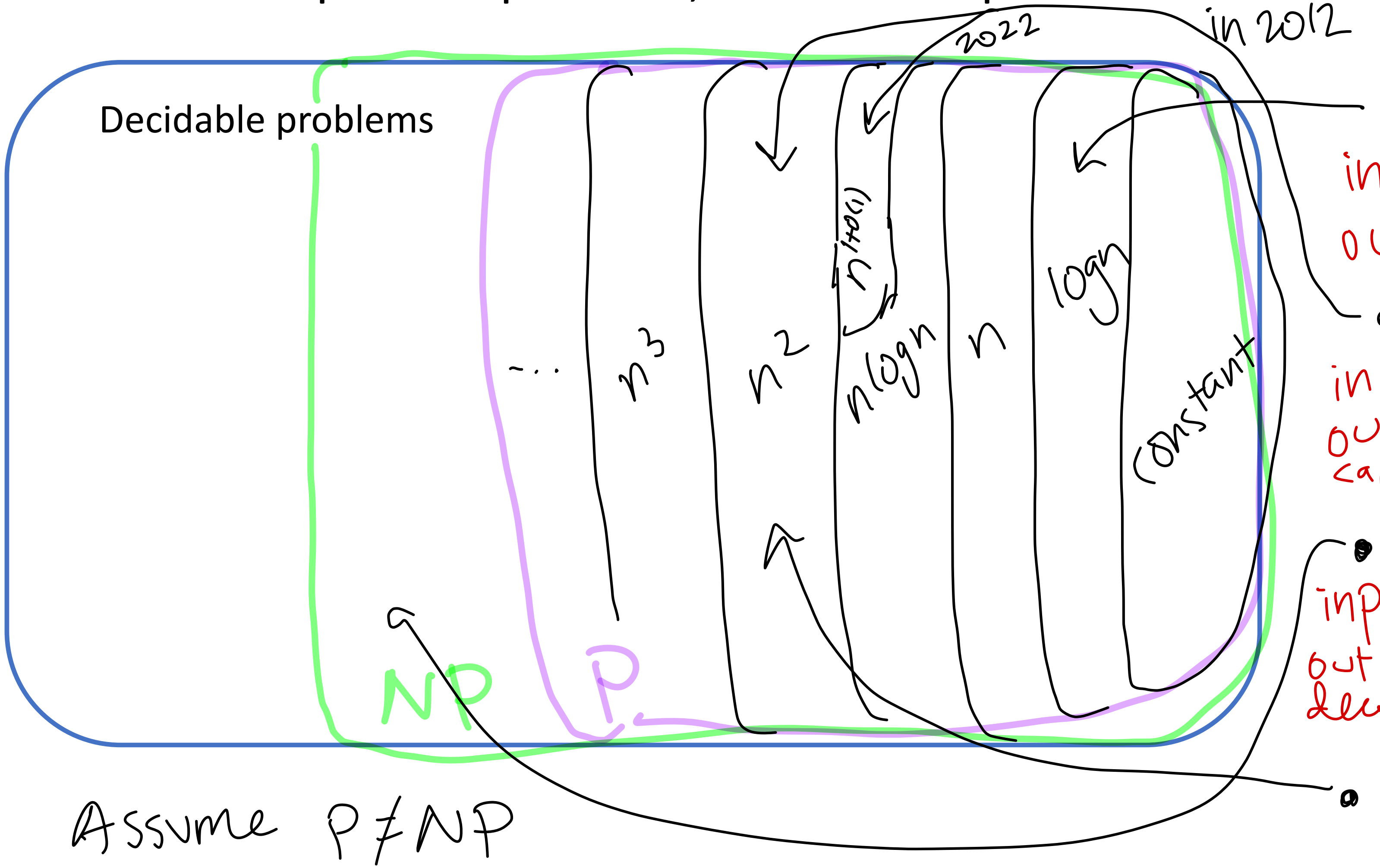

(Decidable)
Example Problems

- searching a sorted list
  input: sorted list, target
  output: whether target in list
- max flow
  input: $G=(V,E)$, edge capacities
  output: biggest flow respecting capacities
- Minimum flow decomposition
  input: acyclic $G=(V,E)$, flow f
  output: smallest set of weighted paths decomposing f
- flow decomposition

Goals for the next three lectures

today: intuitive understanding of LP

next time: formalize

tuesday: algorithms to solve LPs

Maximizing Profit

MT Frisbee Company (MFC) sells two frisbees: The Ripper, and far fancier Ripper Carbon. MFC needs to decide how much of each frisbee it should make to maximize profits.

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1. Rippers yield profit of $10 and Ripper Carbons $30.
2. MFC can sell up to 30 Rippers and 20 Ripper Carbons per day.
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Day's Profit: ?

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Let, $x_1 =$ # of Rippers sold in a day, $x_2 =$ # of Ripper Carbons sold in a day.

Day's Profit: $10x_1 + 30x_2$

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Objective: $10x_1 + 30x_2$

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Objective: $\max 10x_1 + 30x_2$

Subject to: ?

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Objective: $\max 10x_1 + 30x_2$

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Subject to: $x_1 \leq 30$

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Linear Program (LP)

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Subject to: $x_1 \leq 30$
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Linear Program (LP)

Variables:

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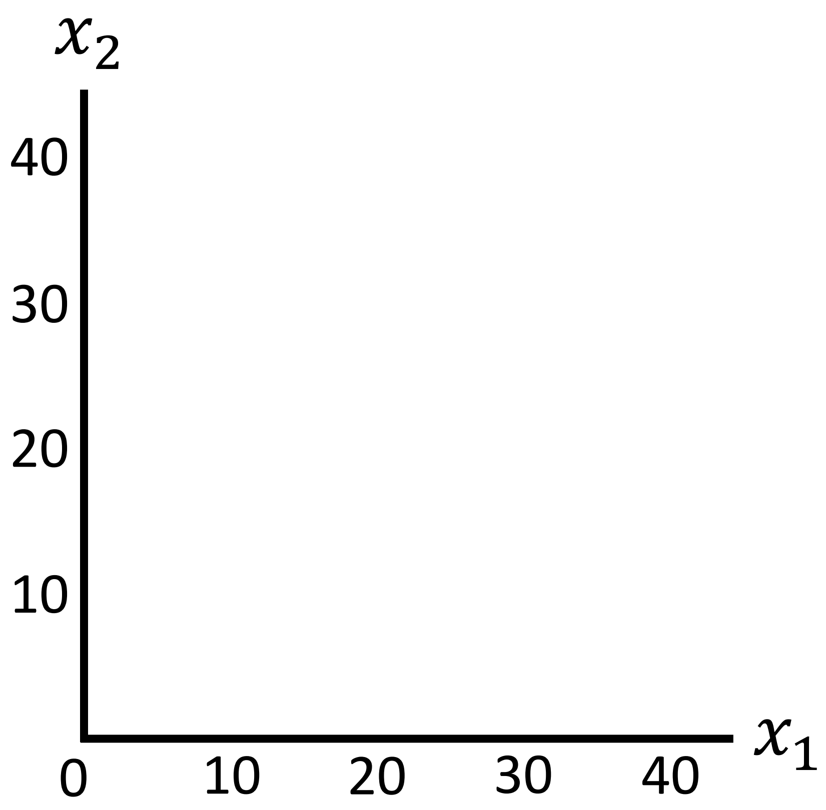

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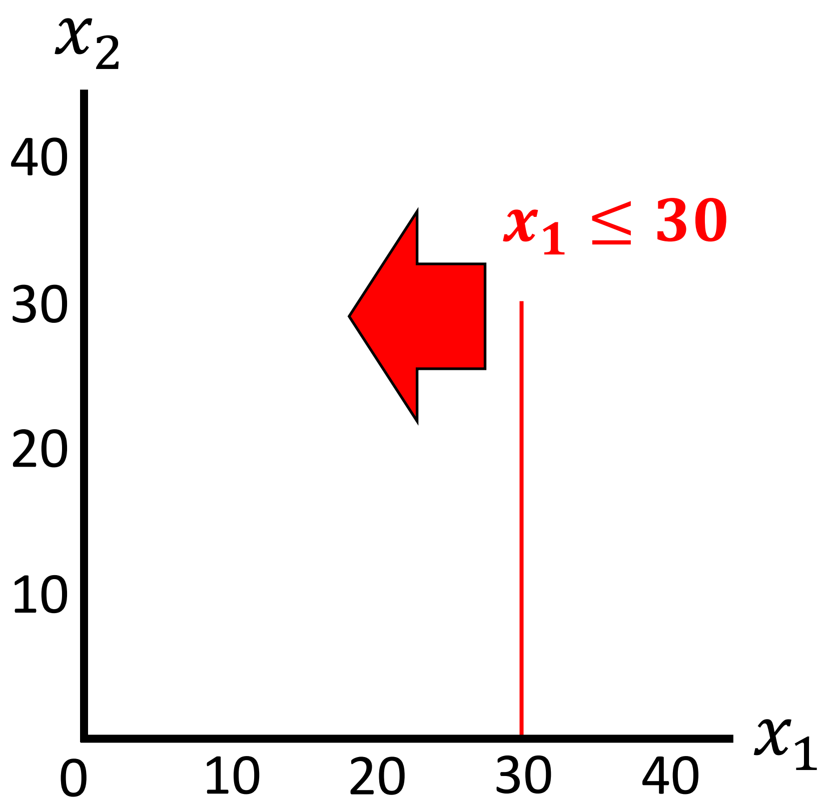

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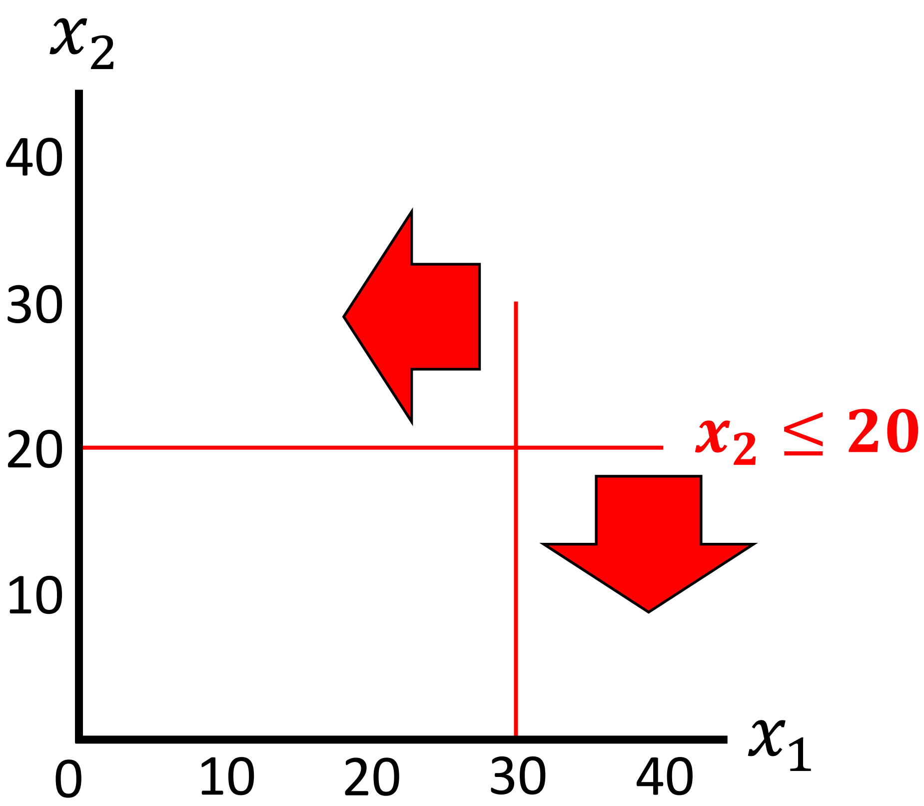

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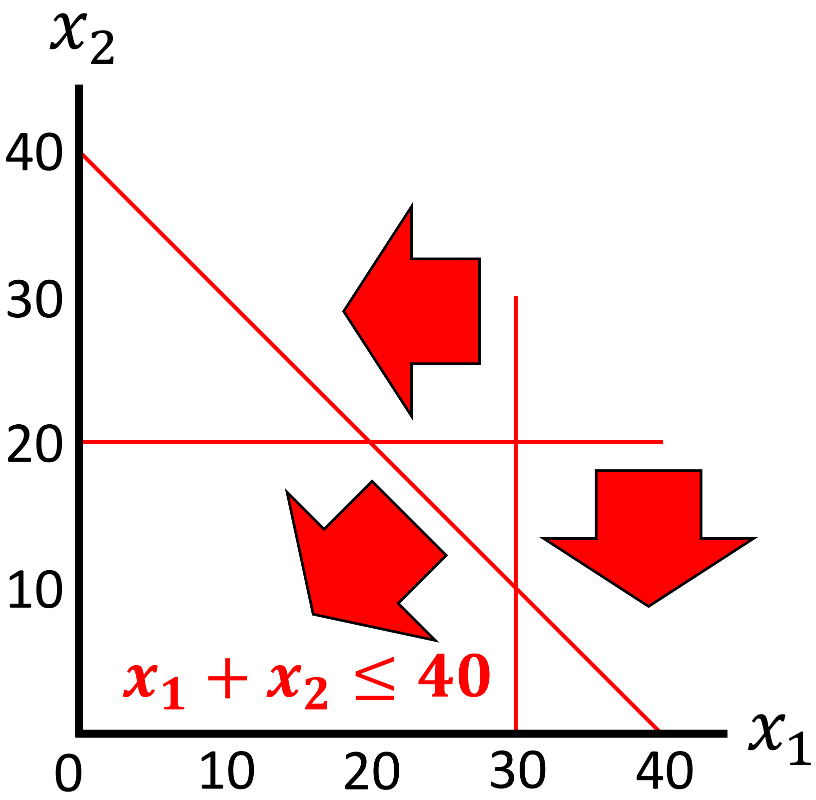

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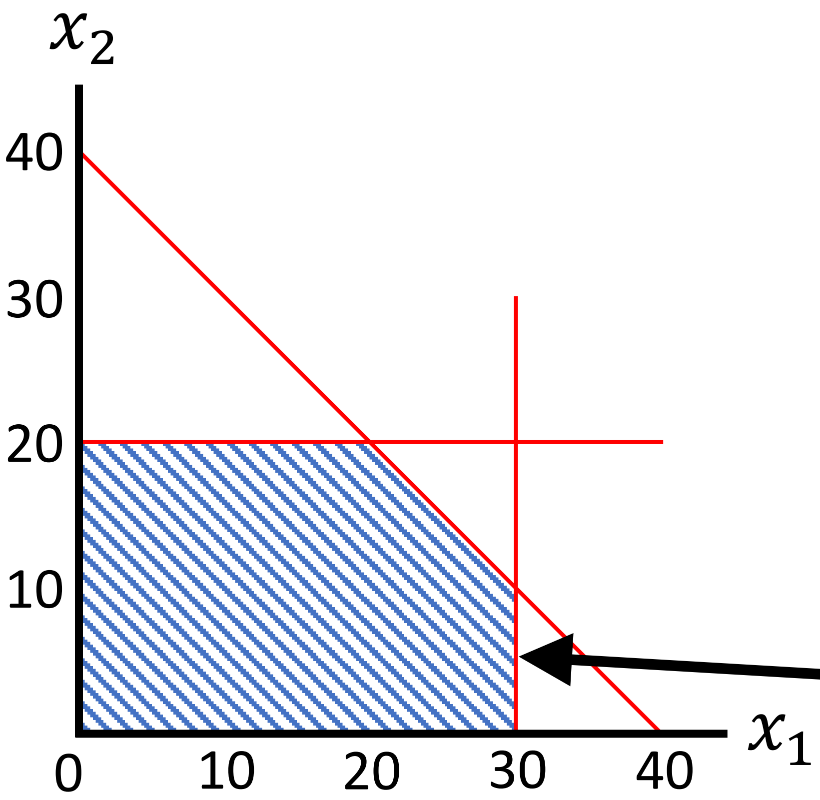
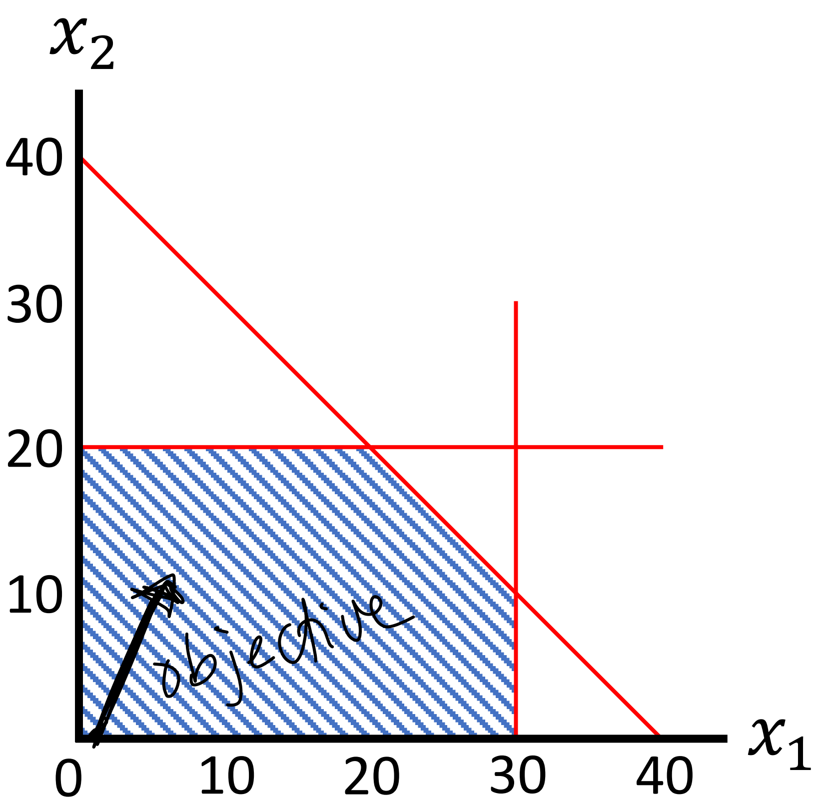


Feasible Region
(area where *all* valid solutions lie)

Maximizing Profit



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What is the optimal value?

Maximizing Profit

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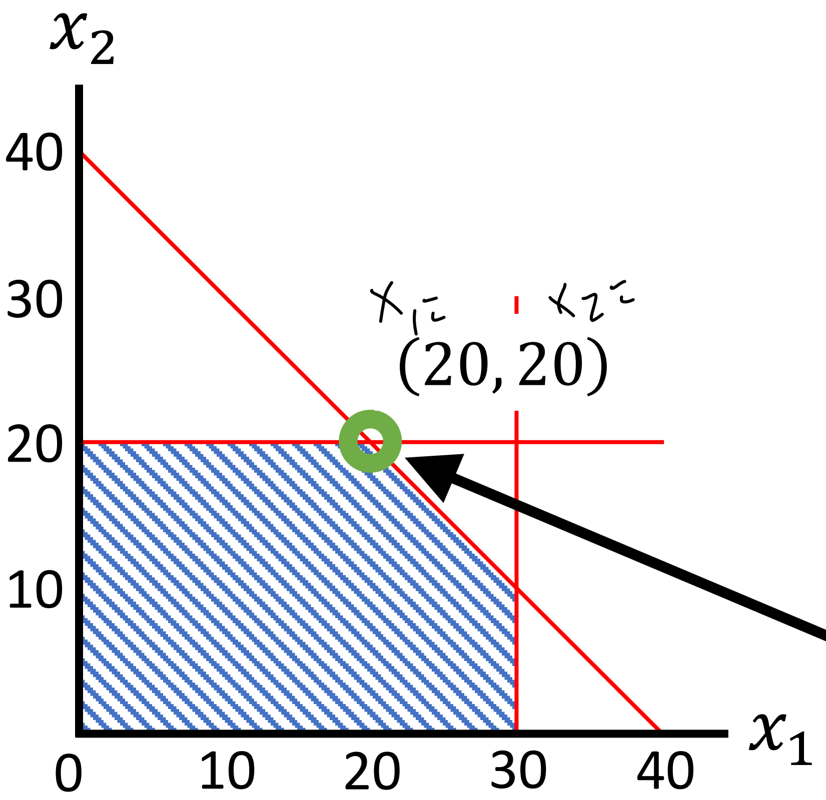

obj $= 10 \times 20 + 30 \times 20 = 800$

Maximizing Profit Modification

MT Frisbee Company (MFC) wants to introduce a third frisbee aimed at kids: Ripper Jr. The Jr yields a profit of $15. Unfortunately, the Ripper and Ripper Jr use the same machine (two hours per frisbee for the Ripper and one hour for the Ripper Jr). There are only 60 machine hours available each day.

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New formulation?

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$x_3 =$ # of Ripper Jrs sold in a day

Objective: $\max 10x_1 + 30x_2 + 15x_3$

Subject to:

~~$x_1 \leq 30$~~

$x_2 \leq 20$

$x_1 + x_2 + x_3 \leq 40$

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What does the feasible region look like now?

Maximizing Profit Modification

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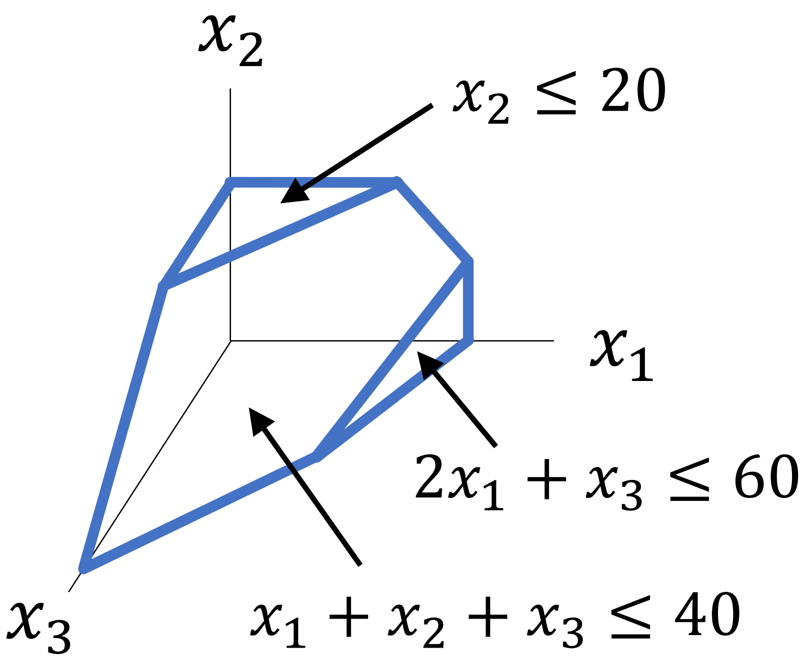

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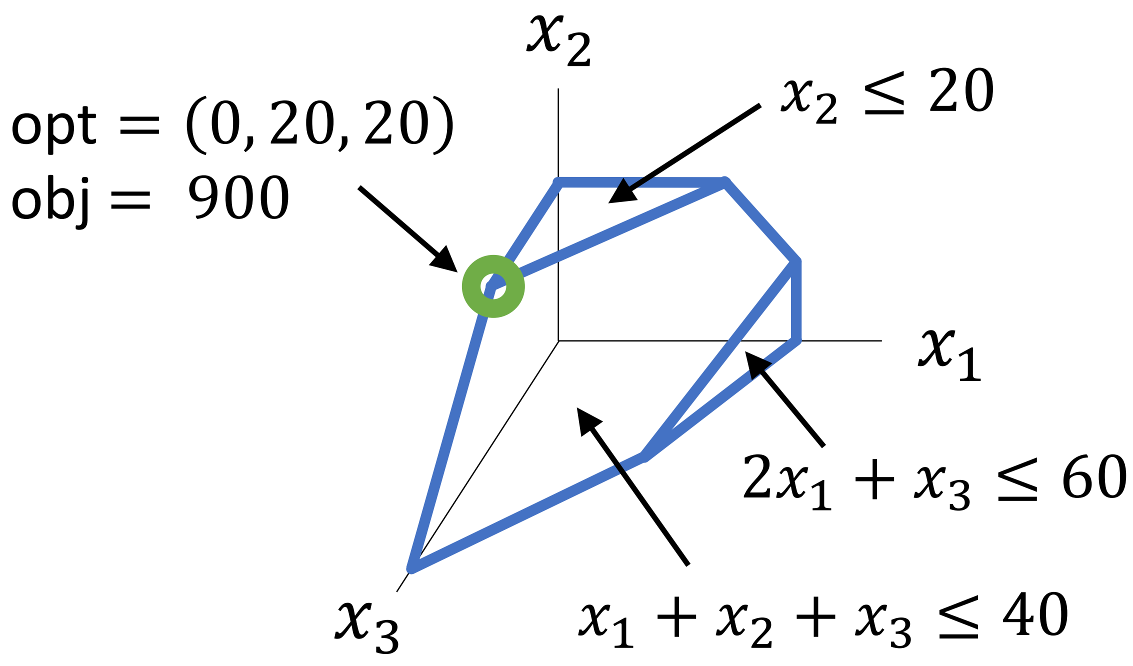

Linear Program (LP)

→ + explanation of meaning

x_1 = # of Rippers sold
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Objective: $\max 10x_1 + 30x_2$

Subject to:
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Issue	**Urban**	**Suburban**	**Rural**
Infrastructure	-2	+5	+3
Gun Control	+8	+2	-5
Farm Subsidies	+0	+0	+10
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Step 1: Make variables.

“What are the decisions that need to be made?”

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Step 2: Make objective.

"What are we trying to maximize or minimize?"

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Objective: $\min x_1 + x_2 + x_3 + x_4$

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Step 3: Make constraints.

"What are the requirements for the solution to be valid?"

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Subject to: $-2x_1 + 8x_2 + 10x_4 \geq 50{,}000$

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Subject to: $-2x_1 + 8x_2 + 10x_4 \geq 50{,}000$
$5x_1 + 2x_2 \geq 100{,}000$
$3x_1 - 5x_2 + 10x_3 - 2x_4 \geq 25{,}000$

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Subject to: $-2x_1 + 8x_2 + 10x_4 \geq 50{,}000$

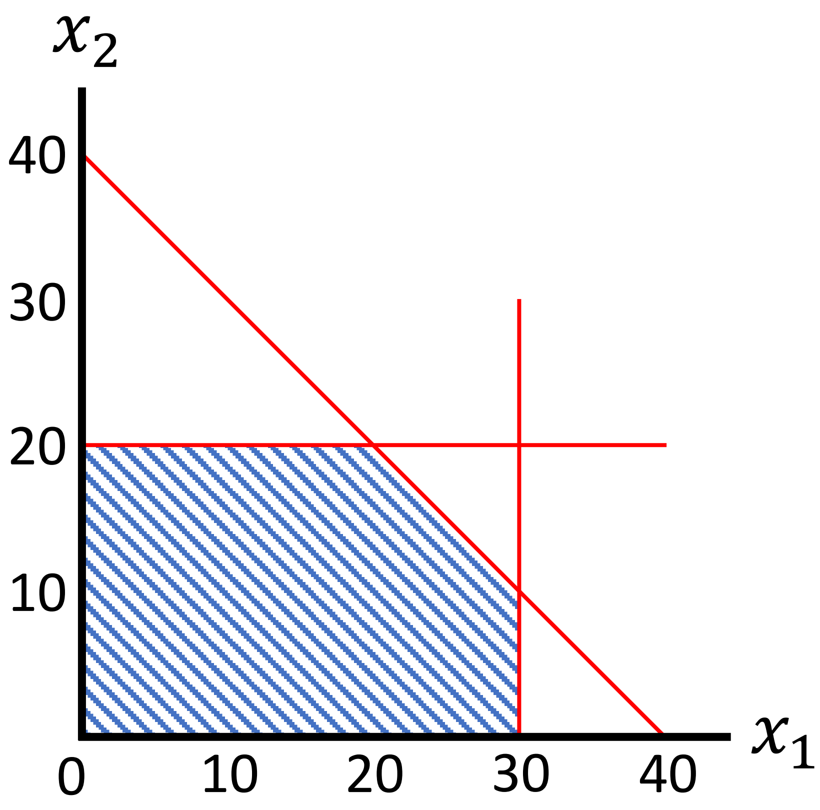
$5x_1 + 2x_2 \geq 100{,}000$

$3x_1 - 5x_2 + 10x_3 - 2x_4 \geq 25{,}000$

$x_1, x_2, x_3, x_4 \geq 0$

Do we need non-negativity constraints?

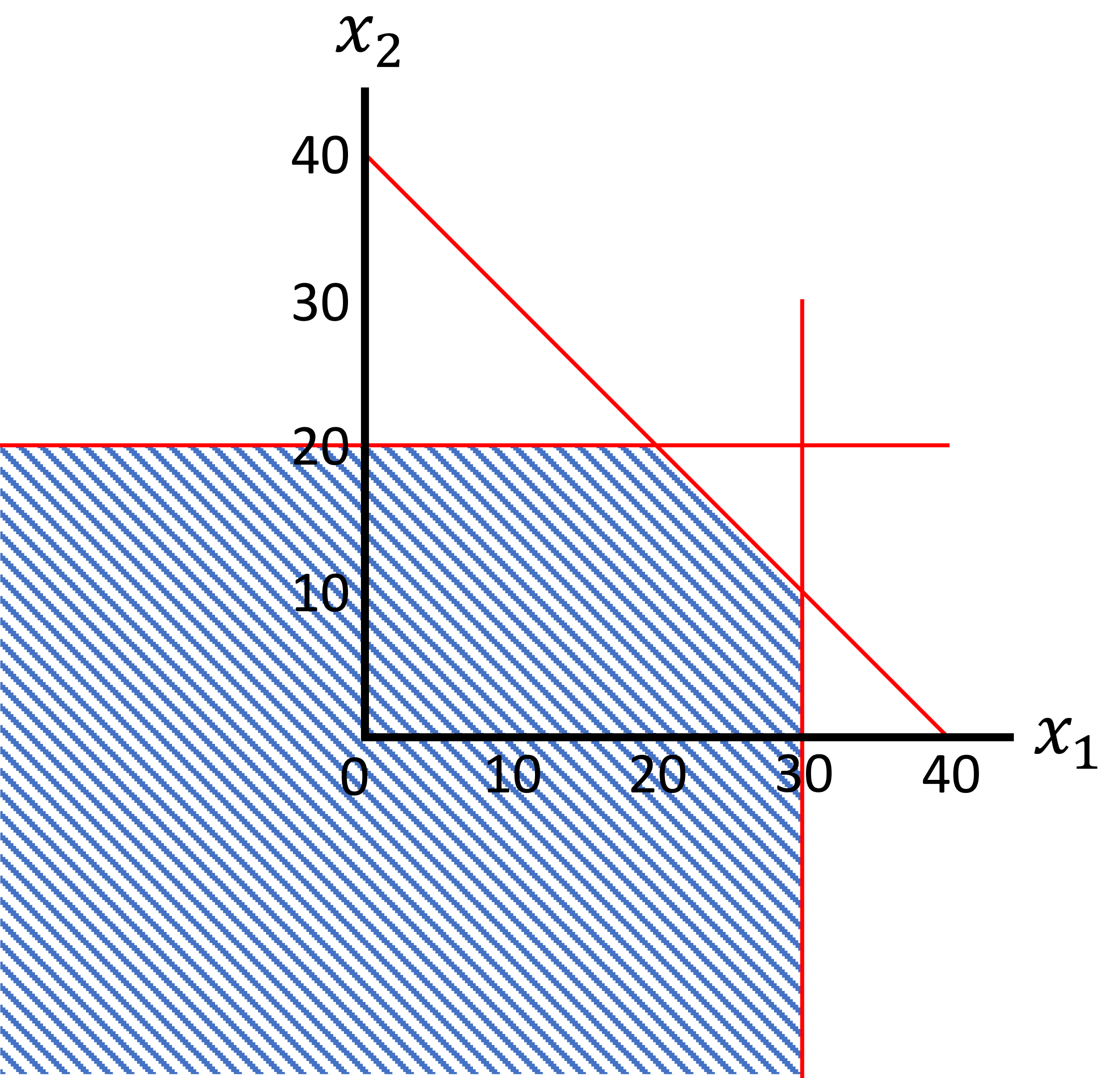
Non-negativity Constraints



Objective: $\max 100x_1 + 300x_2$

Subject to:
$x_1 \leq 30$
$x_2 \leq 20$
$x_1 + x_2 \leq 40$
$x_1, x_2 \geq 0$

Non-negativity Constraints



Objective: $\max 10x_1 + 30x_2$

Subject to:
$x_1 \leq 30$
$x_2 \leq 20$
$x_1 + x_2 \leq 40$
~~$x_1, x_2 \geq 0$~~

Non-negativity Constraints

Objective: $\min x$

Subject to: $x \geq 0$

Optimal Value: ?

Non-negativity Constraints

Objective: $\min x$

Subject to: $x \geq 0$

Optimal Value: $x = 0$

Non-negativity Constraints

Objective: $\min x$

Subject to: ~~$x \geq 0$~~

Optimal Value: ?

Non-negativity Constraints

Objective: $\min x$

Subject to: $x \geq 0$

Optimal Value: $x = -\infty$

A district has an urban area (100,000 voters), suburban area (200,000 voters), and rural area (50,000 voters). A politician decided she needs at least half of the voters in each area to support her. Her campaign has four issues which are popular/unpopular with specific areas. The campaign has estimated the number of voters gained or lost based on each $1 spent advertising an issue. The campaign aims to minimize advertising expenses.

Issue	Urban	Suburban	Rural
Infrastructure	-2	+5	+3
Gun Control	+8	+2	-5
Farm Subsidies	+0	+0	+10
Gasoline Tax	+10	+0	-2

x_1 = \$ spent on infrastructure.
x_2 = \$ spent on gun control.
x_3 = \$ spent on farm subsidies.
x_4 = \$ spent on gasoline tax.

Objective: $\min x_1 + x_2 + x_3 + x_4$

Subject to: $-2x_1 + 8x_2 + 10x_4 \geq 50{,}000$

$5x_1 + 2x_2 \geq 100{,}000$

$3x_1 - 5x_2 + 10x_3 - 2x_4 \geq 25{,}000$

$x_1, x_2, x_3, x_4 \geq 0$

Do we need non-negativity constraints?

A district has an urban area (100,000 voters), suburban area (200,000 voters), and rural area (50,000 voters). A politician decided she needs at least half of the voters in each area to support her. Her campaign has four issues which are popular/unpopular with specific areas. The campaign has estim... e number of voters gained or lost based on each $1 spent advertisi... campaign aims to minimize advertising expenses.

Issue	Urban	Suburban	Rural
Infrastructure	-2	+5	+3
Gun Control	+8	+2	[illegible]
Farm Subsidies	+0	[illegible]	[illegible]
Gasoline Tax	[illegible]	[illegible]	[illegible]

...astructure.
... on gun control.
... $ spent on farm subsidies.
$x_4 = \$$ spent on gasoline tax.

Objective: $\min x_1 + x_2 + x_3 + x_4$

Subject to: $-2x_1 + 8x_2 + 10x_4 \geq 50{,}000$

$5x_1 + 2x_2 \geq 100{,}000$

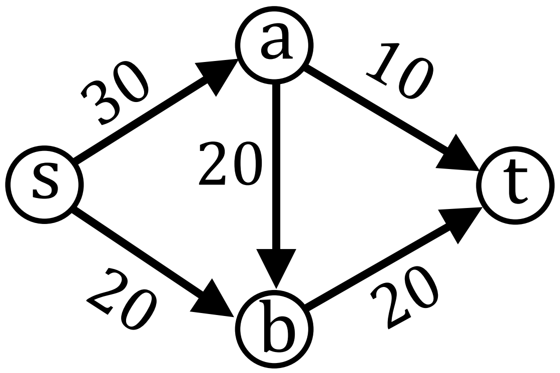
$3x_1 - 5x_2 + 10x_3 - 2x_4 \geq 25{,}000$

$x_1, x_2, x_3, x_4 \geq 0$

Lesson: If the problem calls for implicitly calls for non-negativity constraints, put them in!

Do we ... on-negativity constraints?

Maximum Flow Problem: Suppose we have the flow network below where each edge is labeled with its capacity. Give an LP whose solution is an s-t flow of maximum size.



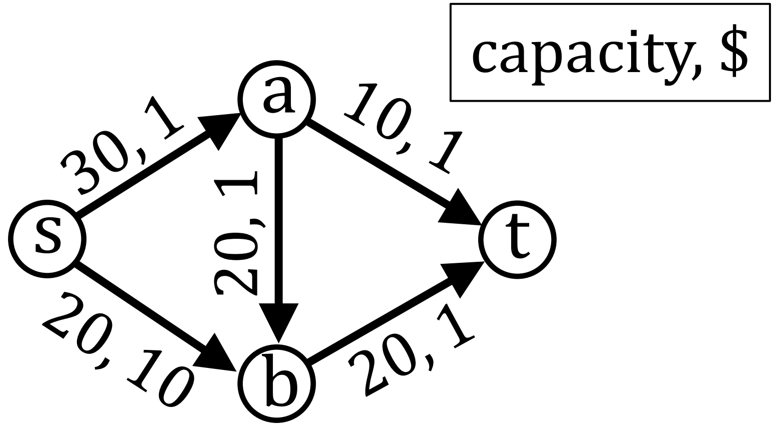
Some notation we've used in the past:

- We denote the capacity of edge s→a as c(s→a). (Note that c(s→a)=30 here.)
- We used f(s→a) to denote the flow on edge s→a
- To write that flow is conserved at node b, we might say

$$\sum_u f(u \to b) = \sum_v f(b \to v)$$

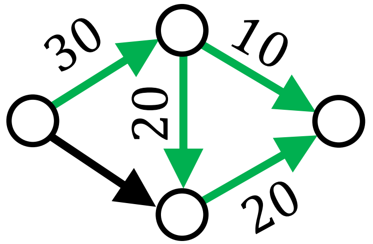
Bonus:
Give the general formulation for a generic graph G=(V,E) with capacities.

Minimum-Cost Flow Problem: Suppose we have a target flow demand d, and a flow network where each edge also has a cost in addition to its capacity. Pushing k flow along edge e incurs the cost $kc(e)$. Find an $s-t$ flow of minimum cost with value d.

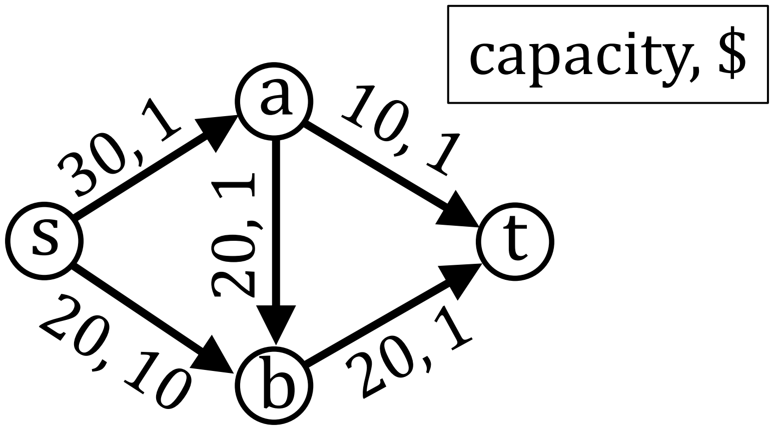


Minimum-Cost Flow Problem: Suppose we have a target flow demand d, and a flow network where each edge also has a cost in addition to its capacity. Pushing k flow along edge e incurs the cost $kc(e)$. Find an $s-t$ flow of minimum cost with value d.

Let $d = 30$



$30 + 20 + 10 + 20 = 80$



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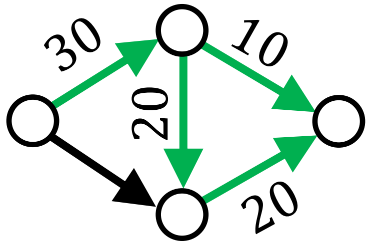
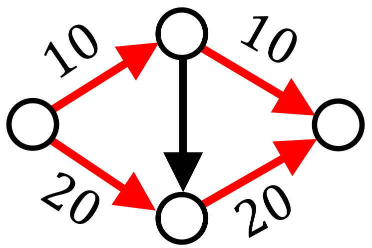
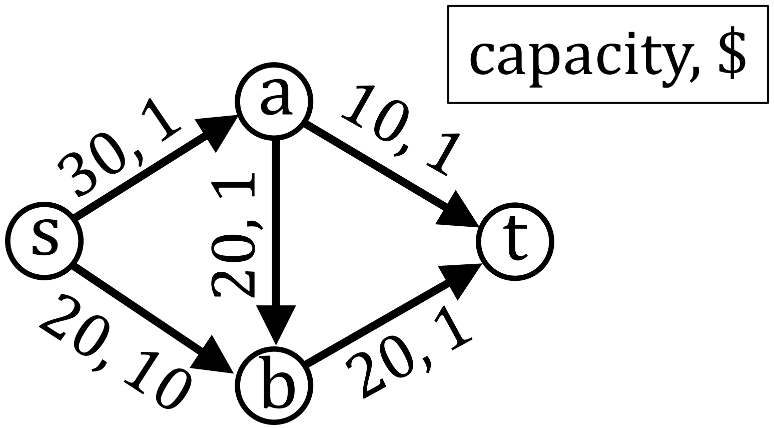


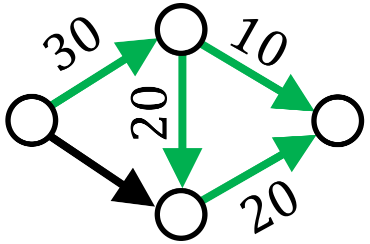


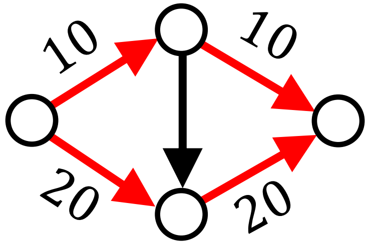


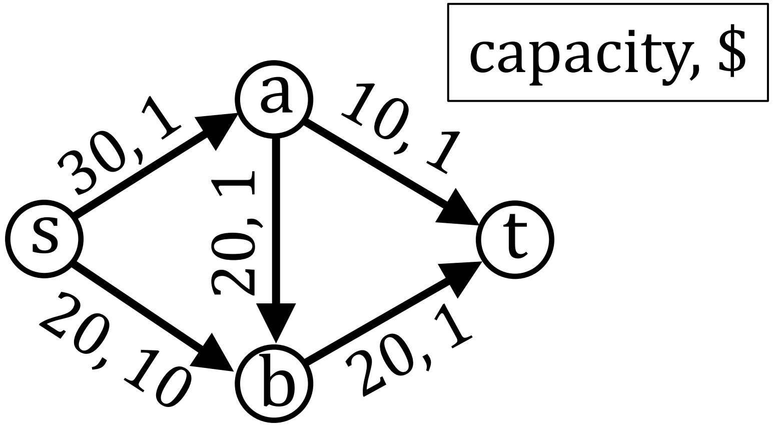
$30 + 20 + 10 + 20 = 80$ $10 + 10 + 200 + 20 = 240$

<u>Minimum-Cost Flow Problem</u>: Suppose we have a target flow demand d, and a flow network where each edge also has a cost in addition to its capacity. Pushing k flow along edge e incurs the cost $kc(e)$. Find an $s-t$ flow of minimum cost with value d.

Let $d = 30$







$30 + 20 + 10 + 20 = 80$ $\quad$ $10 + 10 + 200 + 20 = 240$

Linear Program?