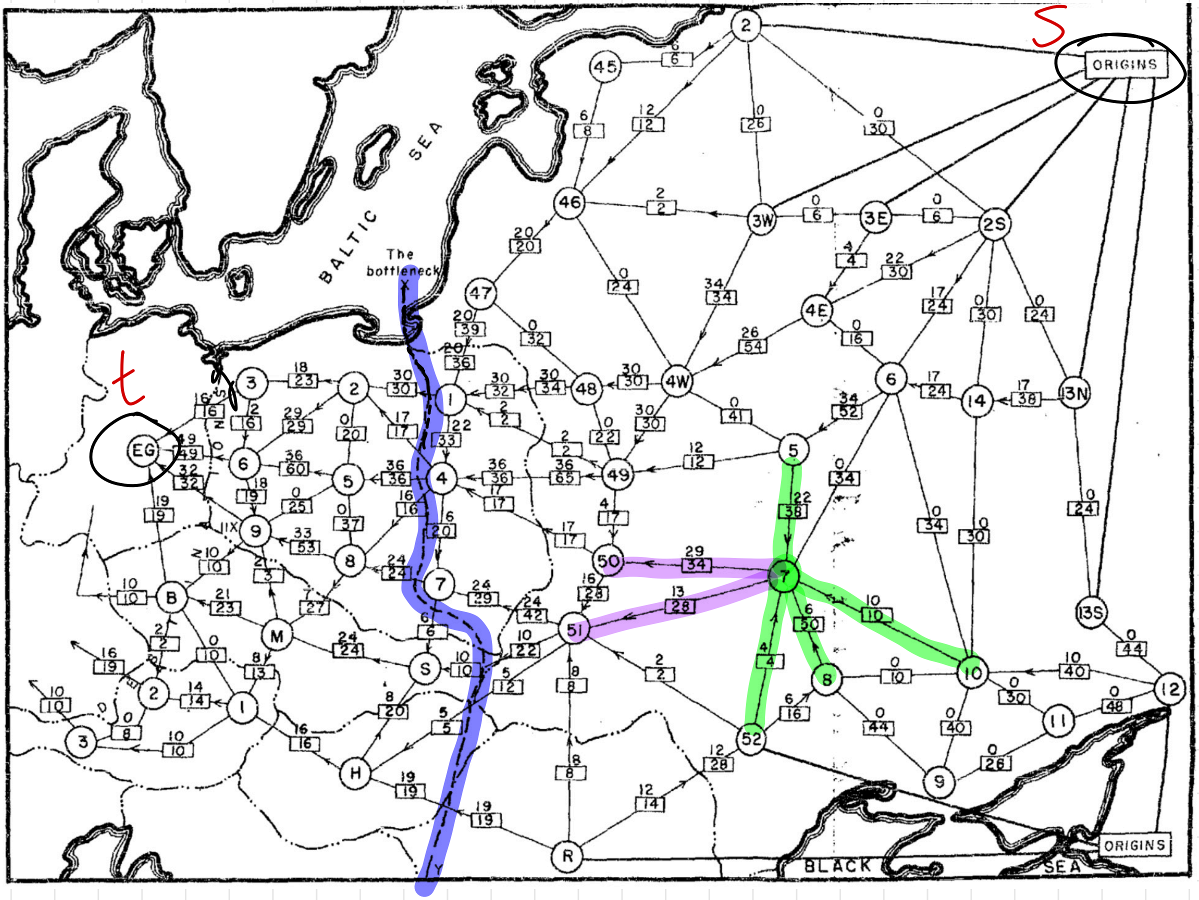
why bother?

If you prove a problem NP-hard,

- if your inputs are small, just solve it
- use an approximation
- use a heuristic → no guarantee of optimality
- try a diff problem formulation or approach
- try to specialize
- use a SAT solver or ILP solver
- prove P=NP


s
ORIGINS
t
EG
BALTIC SEA
The bottleneck
ORIGINS
BLACK SEA

Maximum flow problem:

input: directed graph $G=(V,E)$
with special nodes s, t

source ↗ ↖ target

capacity function $c: E \to \mathbb{R}^{\geq 0}$

output:

flow function $f: E \to \mathbb{R}^{\geq 0}$

such that

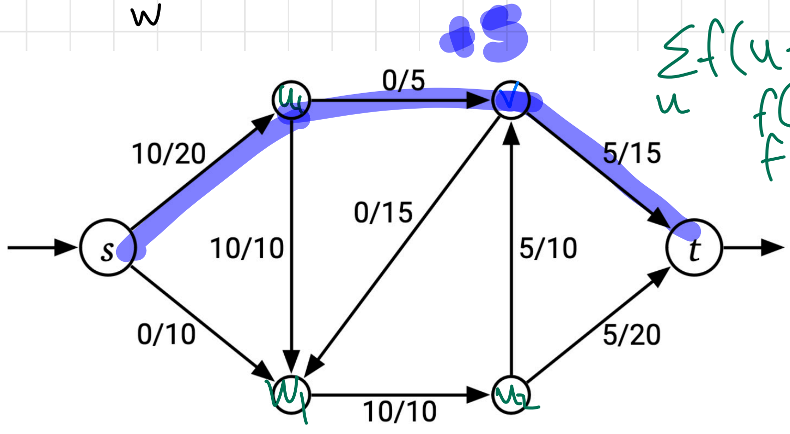
- for all $v \in V \setminus \{s,t\}$ $\quad \sum_u \overset{5}{f(u \to v)} = \sum_w \overset{5}{f(v \to w)}$

  conservation of flow

- for all $e \in E$ $\quad 0 \leq f(e) \leq c(e)$

  feasibility

- $|f| = \sum_w f(s \to w)$ is maximized



$\sum_u f(u \to v) =$
$f(u_1 \to v) + f(u_2 \to v_2)$
$= 0 + 5$
$= 5$

$$\sum_{w} f(v\to w) = f(v\to t) + f(v\to w_1)$$

$$= 5 + 0 = 5$$

① is f a valid flow?

② is f a max flow?

Minimum Cut Problem

input: same as maxflow

output: partition of verts V into S and T

↓

$V = S \cup T$

$S \cap T = \emptyset$

"the capacity of the cut" ↙

Such that $\|S,T\| = \sum_{v\in S}\sum_{w\in T} c(v\to w)$ is minimized.