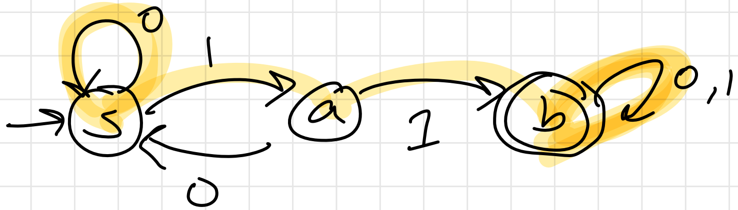


00111001

$$(0+1)^*\, 11\, (0+1)^*$$

Kleene's Theorem:

automatic = regular

proof sketch
- given a DFA, show how to write the equivalent regular exp
- given a regular expression, produce the equivalent DFA

$M = (Q, s, A, \delta)$

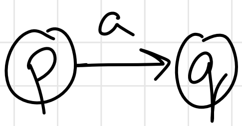
Q – States

$s \in Q$ – start state

$A \subseteq Q$ – accepting states

$\delta : Q \times \Sigma \to Q$ – transition function

$\delta(p, a) = q$



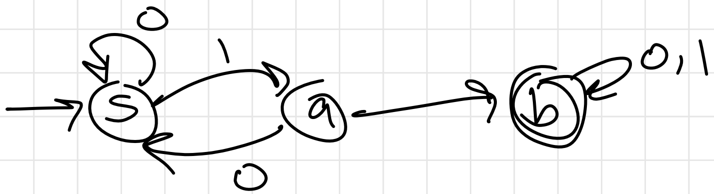
$\delta^*(s, 00111001) = b$

$\delta^* : Q \times \Sigma^* \to Q$

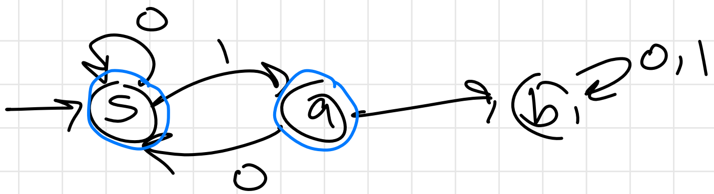
$$\delta^*(q, w) = \begin{cases} q & \text{if } w = \varepsilon \\ \delta^*(\delta(q,a), x) & \text{if } w = ax \end{cases}$$

$L(M) = \{w : M \text{ accepts } w\}$

$= \{w : \delta^*(s,w) \in A\}$



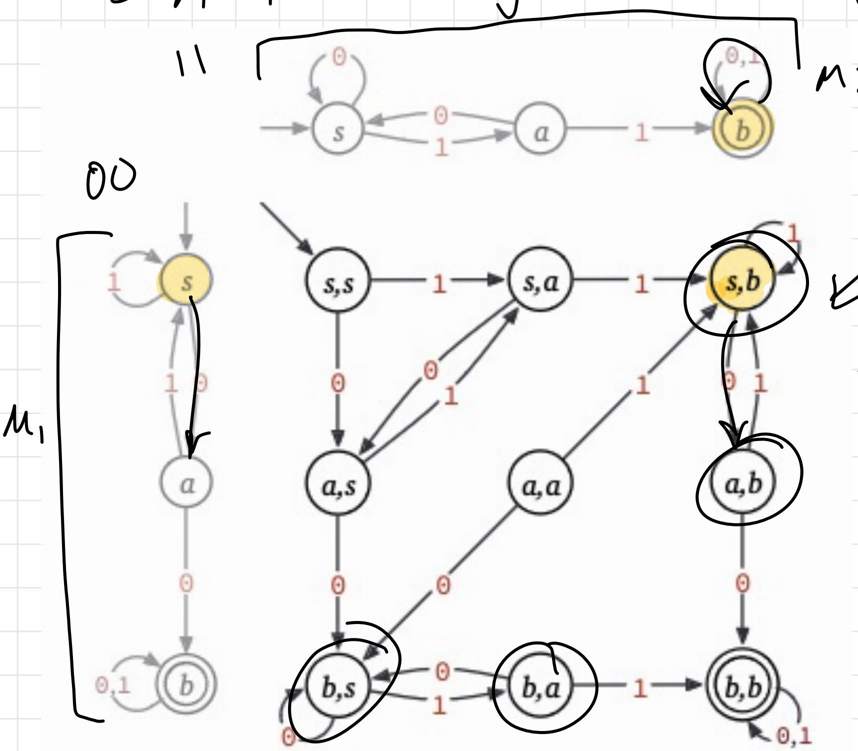
strings containing 11



strings not containing 11

$(0+1)^* 11 (0+1)^*$

DFA for strings containing both 00 and 11



"product construction"

see a 0 (a,b)

Given M_1 and M_2

$M_1 = (Q_1, s_1, A_1, \delta_1)$

$M_2 = (Q_2, s_2, A_2, \delta_2)$

Define M, the product construction, as follows:

$Q = Q_1 \times Q_2 = \{(p,q) : p \in Q_1 \text{ and } q \in Q_2\}$

$s = (s_1, s_2)$

$A = \{(p,q) : p \in A_1 \text{ and } q \in A_2\}$ (and → or / xor / but not)

$\delta((p,q), a) = (\delta_1(p,a), \delta_2(q,a))$

Theorem: $L(M) = L(M_1) \cap L(M_2)$

what is the product construction for

- all strings containing either 00 or 11 (exclusive or)
- all strings containing 00 or 11
- strings containing 00 but not 11

$M_1 \cup M_2$

$M_1 \cap M_2$ ⟹

$M_1 \oplus M_2$

...

Key Lemma: $\delta^*((p,q),w) = (\delta_1^*(p,w), \delta_2^*(q,w))$

for all $p \in Q_1$, $q \in Q_2$, $w \in \Sigma^*$.

Proof Let p, q be arbitrary states, w be an arbitrary string.

Assume that for all x such that $|x| < |w|$, for all $p' \in Q_1$ and q' in Q_2,

$$\delta^*((p',q'),x) = (\delta_1^*(p',x), \delta_2^*(q',x))$$

There are 2 cases:

$w = \varepsilon$

$$\begin{aligned}
\delta^*((p,q),w) &= \delta^*((p,q),\varepsilon) && w=\varepsilon \\
&= (p,q) && \text{def. of } \delta^* \\
&= (\delta_1^*(p,\varepsilon), \delta_2^*(q,\varepsilon)) && \text{def of } \delta_1^*, \delta_2^* \\
&= (\delta_1^*(p,w), \delta_2^*(q,w)) && w=\varepsilon
\end{aligned}$$

$w = ax$

$$\begin{aligned}
\delta^*((p,q),w) &= \delta^*((p,q),ax) && w=ax \\
&= \delta^*(\underline{\delta((p,q),a)},x) && \text{def of } \delta^* \\
&= \delta^*((\underline{\delta_1(p,a)}, \underline{\delta_2(q,a)}),x) && \text{def of } \delta
\end{aligned}$$

$$= \left(\delta_1^*(\delta_1(p,a), x),\ \delta_2^*(\delta_2(q,a), x)\right)$$

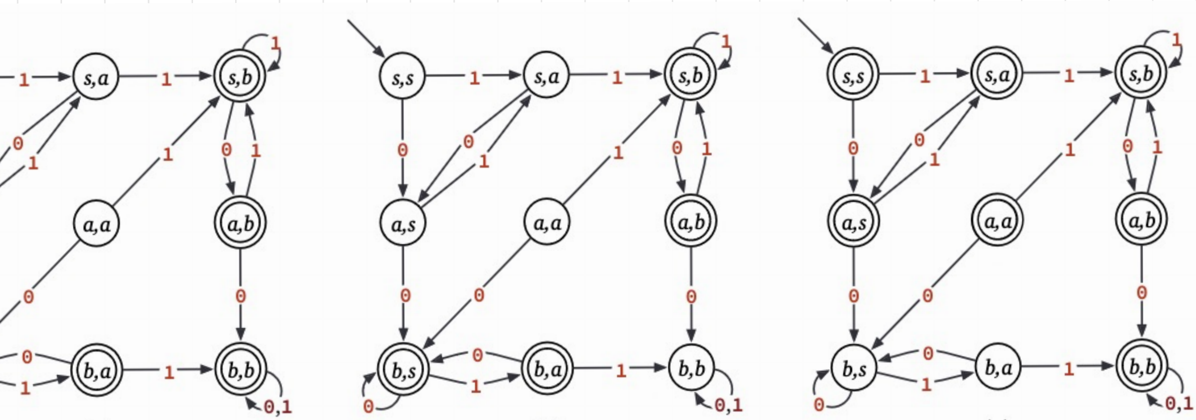
by IH

$$= \left(\delta_1^*(p, ax),\ \delta_2^*(q, ax)\right)$$

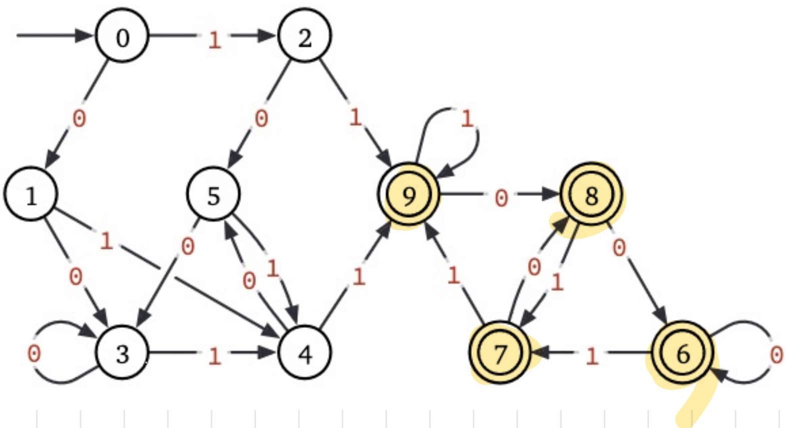
by def of δ_1^* and δ_2^*

$$= \left(\delta_1^*(p, w),\ \delta_2(q, w)\right)$$

$ax = w$



(a) (b) (c)



DFA for accepting strings containing 11

p and q are <u>distinguishable</u> $\Leftrightarrow$

for some string w

$\delta^*(p,w) \in A$ and $\delta^*(q,w) \notin A$

or vice versa