From last time…

MT Frisbee Company (MFC) wants to introduce a third frisbee aimed at kids: Ripper Jr. The Jr yields a profit of $15. Unfortunately, the Ripper and Ripper Jr use the same machine (two hours per frisbee for the Ripper and one hour for the Ripper Jr). There are only 60 machine hours available each day.

$x_1 =$ # of Rippers sold in a day
$x_2 =$ # of Ripper Carbons sold in a day
$x_3 =$ # of Ripper Jrs sold in a day

real-valued variables

Objective: $\max 10x_1 + 30x_2 + 15x_3$

couldn't have $x_1 x_2$

Subject to: $x_2 \leq 20$

$x_1 + x_2 + x_3 \leq 40$

$2x_1 + x_3 \leq 60$

$\leq$, $\geq$, $=$

From last time…

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Objective: $\max 10x_1 + 30x_2 + 15x_3$
Subject to: $x_2 \leq 20$
$x_1 + x_2 + x_3 \leq 40$
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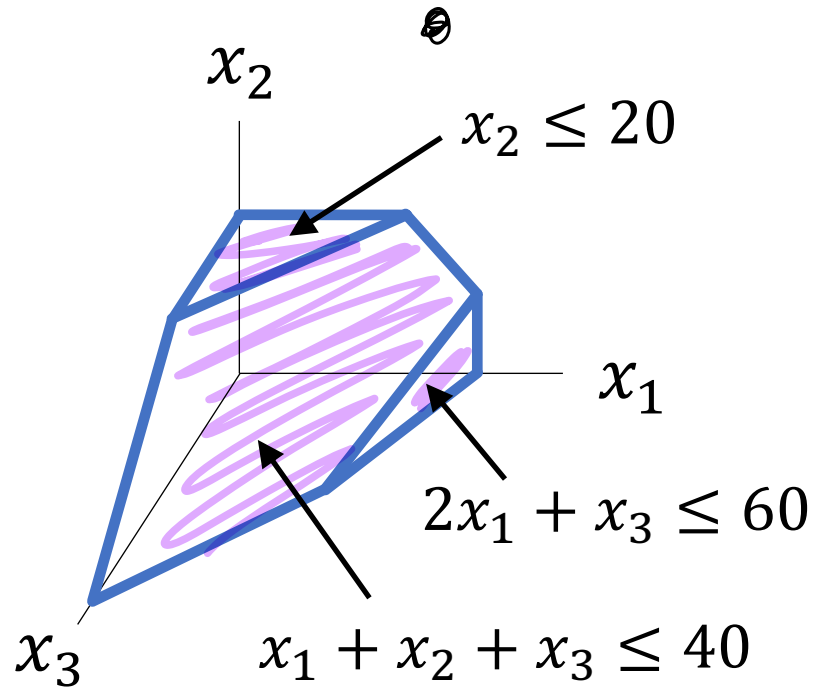
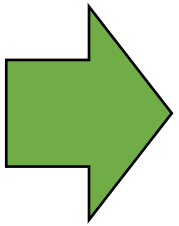


Canonical

LP Standard Form

vectors c, b
matrix A

Objective: $\max 10x_1 + 30x_2 + 15x_3$
Subject to: $x_2 \leq 20$
$x_1 + x_2 + x_3 \leq 40$
$2x_1 + x_3 \leq 60$
$x_1, x_2, x_3 \geq 0$



Objective: max $c^T x$
Subject to: $A x \leq b$
$x \geq 0$

what is x? c? b? A? what's the T?

Canonical

LP Standard Form

max $c \cdot x$
s.t. $Ax \leq b$
$x \geq 0$

Objective: max $c^T x$
Subject to: $Ax \leq b$
$x \geq 0$

Objective: $\max 10x_1 + 30x_2 + 15x_3$
Subject to:
$x_2 \leq 20$
$x_1 + x_2 + x_3 \leq 40$
$2x_1 + x_3 \leq 60$
$x_1, x_2, x_3 \geq 0$

Objective: $\max \begin{bmatrix} 10 & 30 & 15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ (c^T, x)

Subject to: $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \leq \begin{bmatrix} 20 \\ 40 \\ 60 \end{bmatrix}$ (A, b)

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

LP Standard Form

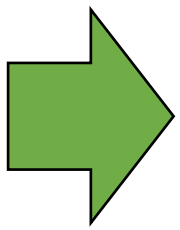
Objective: $\max 10x_1 + 30x_2$

Subject to: $x_1 \leq 30$

$x_2 \leq 20$

$x_1 + x_2 \leq 40$

$x_1, x_2 \geq 0$



Objective: max $c^T x$

Subject to: $A x \leq b$

$x \geq 0$

$$\begin{pmatrix} 10 \\ 30 \end{pmatrix} \quad \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq \begin{pmatrix} 30 \\ 20 \\ 40 \end{pmatrix}$$

Every LP can be turned into standard form.

1.

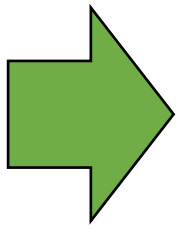
2.

3.

4.

LP Standard Form

Objective: $\max 100x_1 + 300x_2$
Subject to: $x_1 \leq 30$
$x_2 \leq 20$
$x_1 + x_2 \leq 40$
$x_1, x_2 \geq 0$



$\begin{pmatrix}100\\300\end{pmatrix}$ $\begin{pmatrix}x_1\\x_2\end{pmatrix}$

Objective: max $c^T x$
Subject to: $A x \leq b$
$x \geq 0$

$\begin{pmatrix}1 & 0\\0 & 1\\1 & 1\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} \leq \begin{pmatrix}30\\20\\40\end{pmatrix}$

Every LP can be turned into standard form.

1. Minimization → Maximization: ?
   $\min c^T x \longrightarrow \max -c^T x$
2. $\geq$ Constraints → $\leq$: $x_1 + x_2 \geq 5 \Rightarrow -x_1 - x_2 \leq -5$
   multiply by -1 :
3. Equality Constraints → $\leq$: e.g., $x_1 + 3x_2 = 20$ how to make equivalent $\leq$ constraints?
   hint: make 2 constraints + use #2
4. Unrestricted sign $x_1 \rightarrow x_1 \geq 0$:

LP Standard Form

Objective: $\max 100x_1 + 300x_2$
Subject to: $x_1 \leq 30$
$x_2 \leq 20$
$x_1 + x_2 \leq 40$
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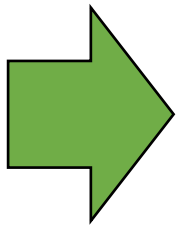


Objective: max $c^T x$
Subject to: $A x \leq b$
$x \geq 0$

$$\begin{pmatrix} 100 \\ 300 \end{pmatrix} \quad \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

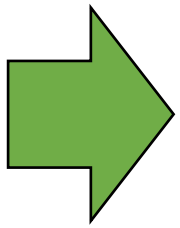
$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq \begin{pmatrix} 30 \\ 20 \\ 40 \end{pmatrix}$$

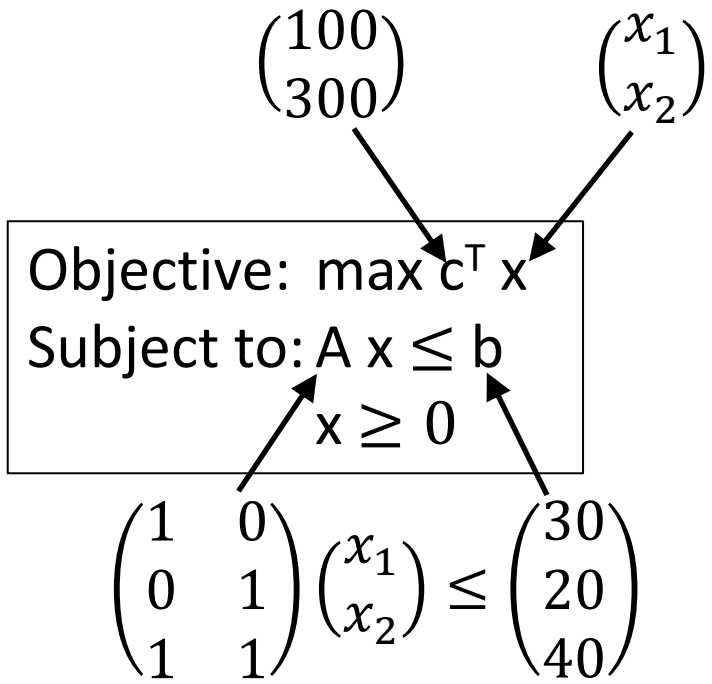
Every LP can be turned into standard form.

1. Minimization → Maximization: Multiply objective coefficients by -1.
   $\min \alpha x_1 + \beta x_2 \rightarrow \max -\alpha x_1 - \beta x_2$
2. $\geq$ Constraints → $\leq$: ?
3. Equality Constraints → $\leq$:
4. Unrestricted sign $x_1 \rightarrow x_1 \geq 0$:

LP Standard Form

Objective: $\max 100x_1 + 300x_2$
Subject to: $x_1 \leq 30$
$x_2 \leq 20$
$x_1 + x_2 \leq 40$
$x_1, x_2 \geq 0$





$$\begin{pmatrix} 100 \\ 300 \end{pmatrix} \quad \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq \begin{pmatrix} 30 \\ 20 \\ 40 \end{pmatrix}$$

Every LP can be turned into standard form.

1. Minimization → Maximization: Multiply objective coefficients by -1.
   $\min \alpha x_1 + \beta x_2 \rightarrow \max -\alpha x_1 - \beta x_2$
2. $\geq$ Constraints → $\leq$: Negate inequality.
   $x_1 + x_2 \geq \alpha \rightarrow -x_1 - x_2 \leq -\alpha$
3. Equality Constraints → $\leq$: ?

4. Unrestricted sign $x_1 \rightarrow x_1 \geq 0$:

LP Standard Form

Objective: $\max 100x_1 + 300x_2$
Subject to: $x_1 \leq 30$
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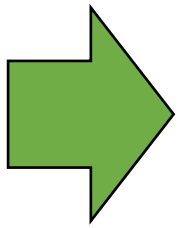

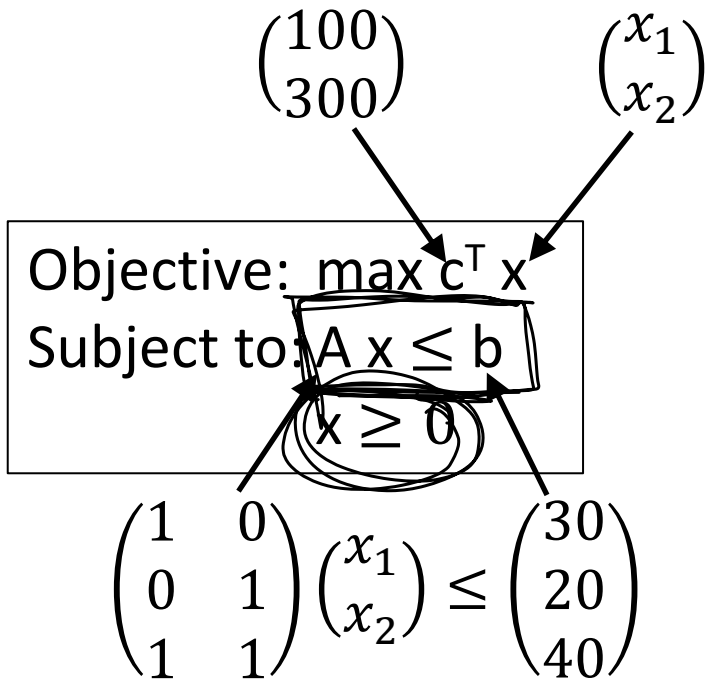

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1. Minimization → Maximization: Multiply objective coefficients by -1.
   $\min \alpha x_1 + \beta x_2 \rightarrow \max -\alpha x_1 - \beta x_2$
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   $x_1 + x_2 \geq \alpha \rightarrow -x_1 - x_2 \leq -\alpha$
3. Equality Constraints → $\leq$: Introduce $\geq$ and $\leq$ constraints.
   $x_1 + x_2 = \alpha \rightarrow x_1 + x_2 \geq \alpha$ and $x_1 + x_2 \leq \alpha$
4. Unrestricted sign $x_1 \rightarrow x_1 \geq 0$: ?

LP Standard Form

Objective: $\max 100x_1 + 300x_2$
Subject to: $x_1 \leq 30$
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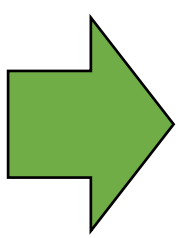

$\begin{pmatrix}100\\300\end{pmatrix}$ $\begin{pmatrix}x_1\\x_2\end{pmatrix}$

Objective: max $c^T x$
Subject to: $A x \leq b$
$x \geq 0$

$\begin{pmatrix}1 & 0\\0 & 1\\1 & 1\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} \leq \begin{pmatrix}30\\20\\40\end{pmatrix}$

Every LP can be turned into standard form.

1. Minimization → Maximization: Multiply objective coefficients by -1.
   $\min \alpha x_1 + \beta x_2 \rightarrow \max -\alpha x_1 - \beta x_2$
2. $\geq$ Constraints → $\leq$: Negate inequality.
   $x_1 + x_2 \geq \alpha \rightarrow -x_1 - x_2 \leq -\alpha$
3. Equality Constraints → $\leq$: Introduce $\geq$ and $\leq$ constraints.
   $x_1 + x_2 = \alpha \rightarrow x_1 + x_2 \geq \alpha$ and $x_1 + x_2 \leq \alpha$
4. Unrestricted sign $x_1 \rightarrow x_1 \geq 0$: Introduce $x_1' \geq 0$, and $x_1'' \geq 0$.
   $x_1 + x_2 \leq \alpha \rightarrow x_1' - x_1'' + x_2 \leq \alpha$

$x_1 = x_1' - x_1'' = 50 - 100 = -50$

$x_1'' = 100, \; x_1' = 50$

Primal and dual linear programs

Maximizing Profit Modification

$x_1 =$ # of Rippers sold in a day
$x_2 =$ # of Ripper Carbons sold in a day
$x_3 =$ # of Ripper Jrs sold in a day

Objective:	$\max 10x_1 + 30x_2 + 15x_3$	
Subject to:	$x_2 \leq 20$	A
	$x_1 + x_2 + x_3 \leq 40$	B
	$2x_1 + x_3 \leq 60$	C
	$x_1, x_2, x_3 \geq 0$	D

Maximizing Profit Modification

$x_1 =$ # of Rippers sold in a day
$x_2 =$ # of Ripper Carbons sold in a day
$x_3 =$ # of Ripper Jrs sold in a day

Objective: $\max 10x_1 + 30x_2 + 15x_3$

Subject to:

$x_2 \leq 20$	A
$x_1 + x_2 + x_3 \leq 40$	B
$2x_1 + x_3 \leq 60$	C
$x_1, x_2, x_3 \geq 0$	D

Linear combinations of constraints are also valid constraints!

2A : $2x_2 \leq 40$

A + B : $x^2 \leq 20$

$+ \quad x_1 + x_2 + x_3 \leq 40$

$x_1 + 2x_2 + x_3 \leq 60$

30A : $30x_2 \leq 600$

Maximizing Profit Modification

x_1 = # of Rippers sold in a day
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Objective:	$\max 10x_1 + 30x_2 + 15x_3$	
Subject to:	$x_2 \leq 20$	A
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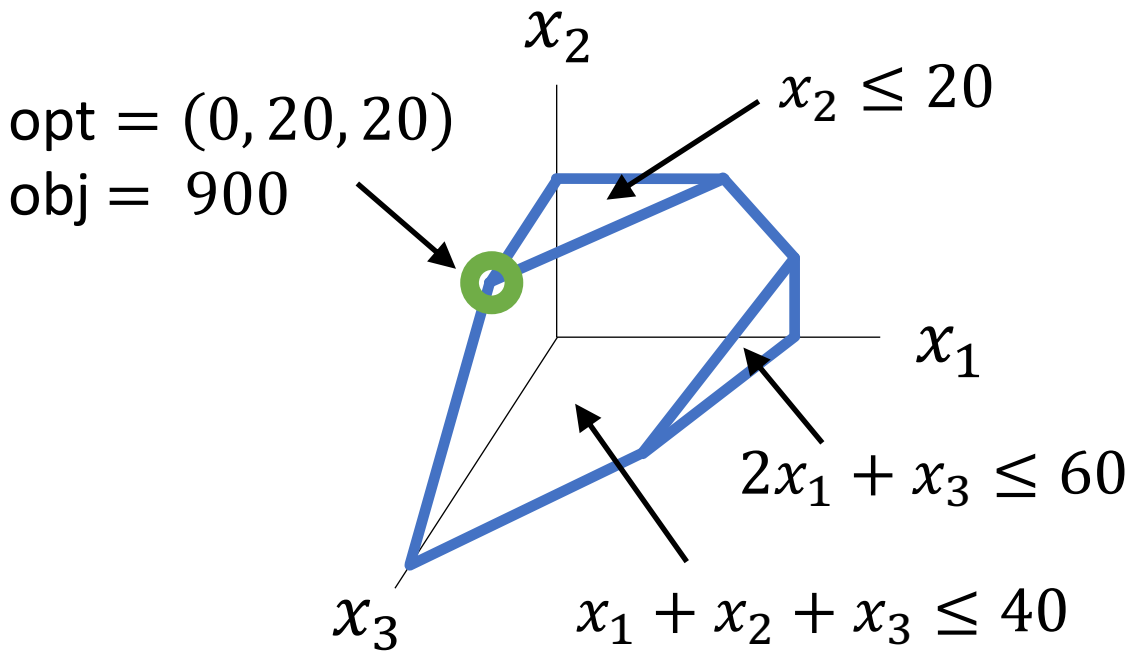
	$2x_1 + x_3 \leq 60$	C
	$x_1, x_2, x_3 \geq 0$	D



Maximizing Profit Modification

x_1 = # of Rippers sold in a day
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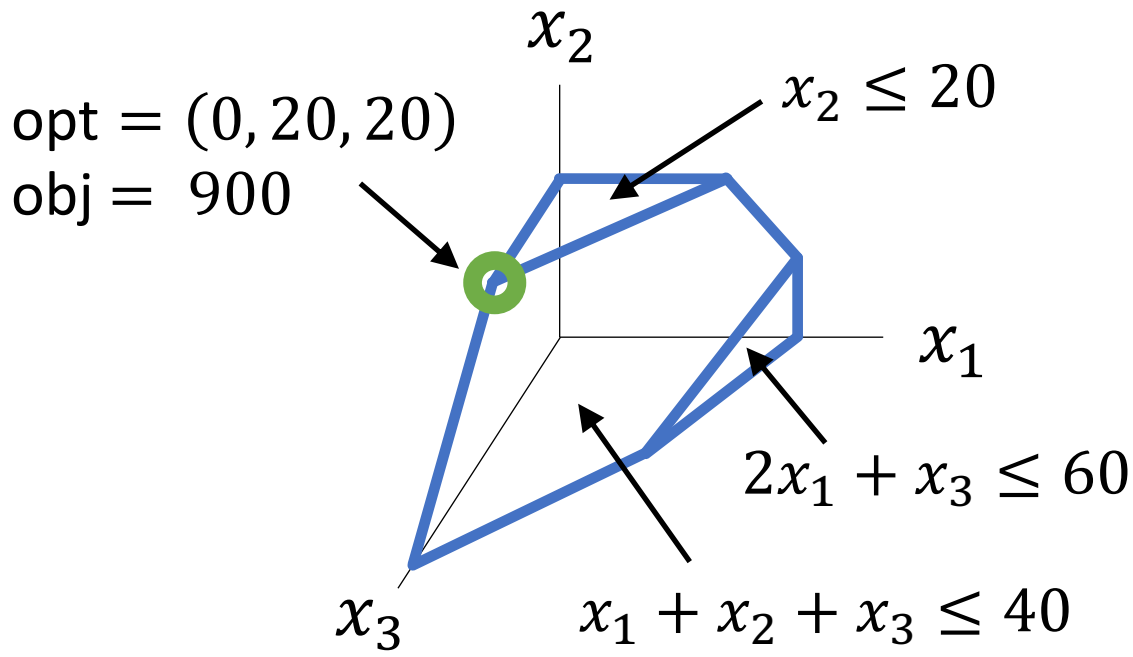
Subject to:	$x_2 \leq 20$	A
	$x_1 + x_2 + x_3 \leq 40$	B
	$2x_1 + x_3 \leq 60$	C
	$x_1, x_2, x_3 \geq 0$	D



$15x_2 \leq 300$ $15x_1 + 15x_2 + 15x_3 \leq 600$

15(Constraint_A) + 15(Constraint_B) $\Longrightarrow 15x_1 + 30x_2 + 15x_3 \leq 900$

Maximizing Profit Modification

x_1 = # of Rippers sold in a day
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x_3 = # of Ripper Jrs sold in a day

Objective:	$\max 10x_1 + 30x_2 + 15x_3$	
Subject to:	$x_2 \leq 20$	A
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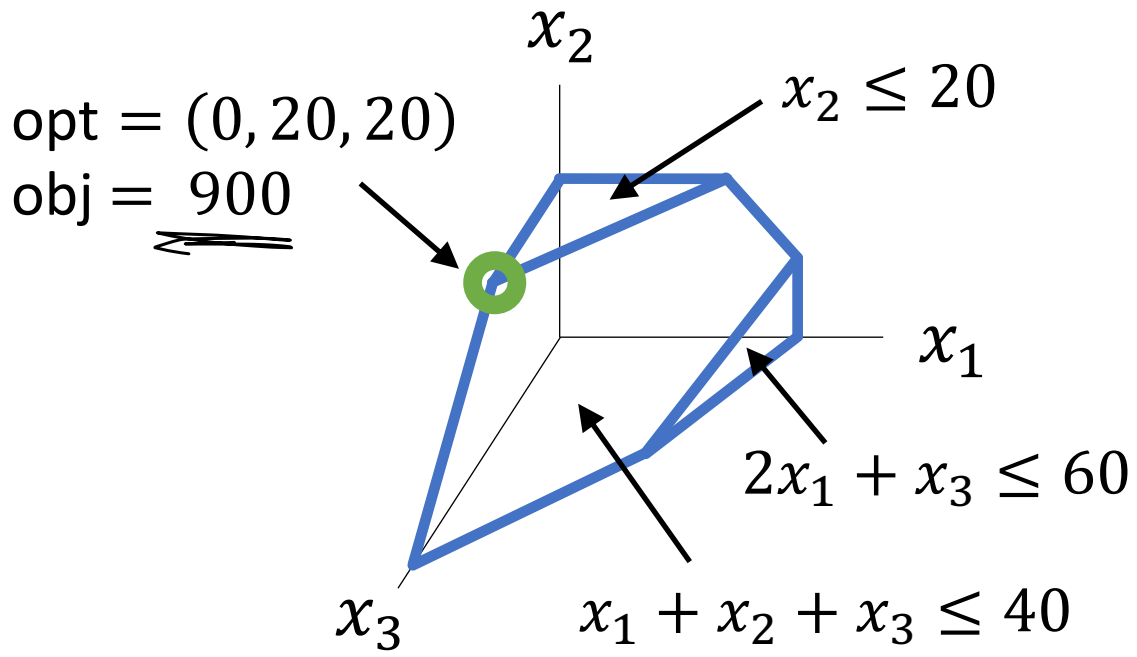


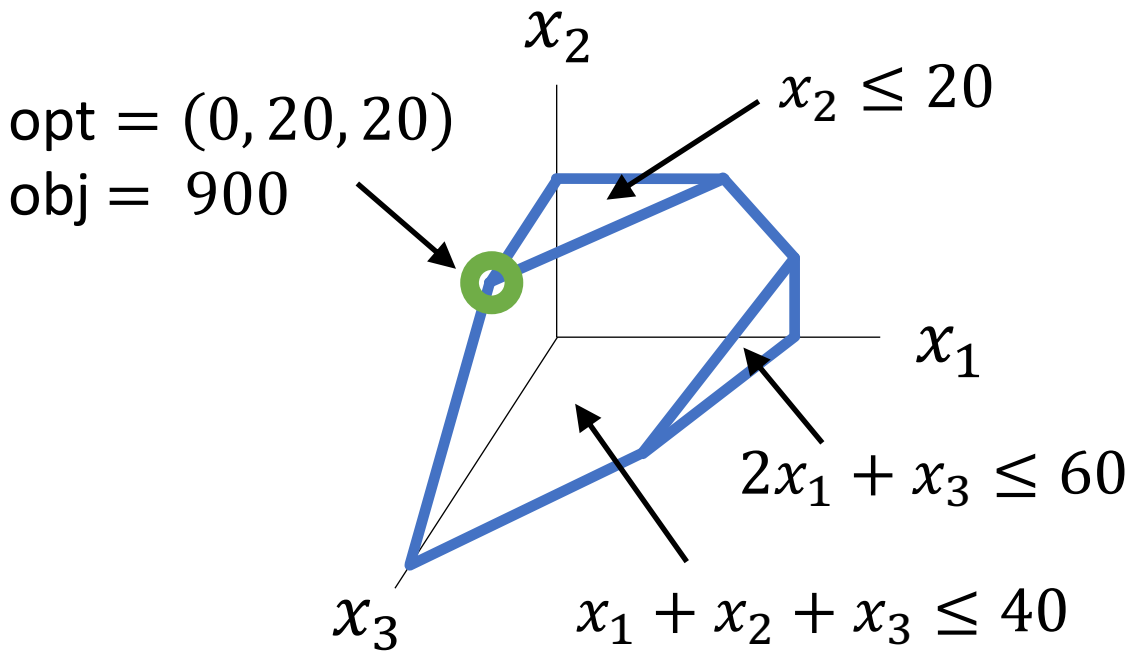
15(Constraint_A) + 15(Constraint_B) $\Longrightarrow 15x_1 + 30x_2 + 15x_3 \leq 900$

$\Longrightarrow 10x_1 + 30x_2 + 15x_3 \leq 900$

Maximizing Profit Modification

x_1 = # of Rippers sold in a day
x_2 = # of Ripper Carbons sold in a day
x_3 = # of Ripper Jrs sold in a day

Objective:	$\max 10x_1 + 30x_2 + 15x_3$	
Subject to:	$x_2 \leq 20$	A
	$x_1 + x_2 + x_3 \leq 40$	B
	$2x_1 + x_3 \leq 60$	C
	$x_1, x_2, x_3 \geq 0$	D



15(Constraint_A) + 15(Constraint_B) $\Longrightarrow 15x_1 + 30x_2 + 15x_3 \leq 900$

$\Longrightarrow 10x_1 + 30x_2 + 15x_3 \leq 900$

Maximizing Profit Modification

x_1 = # of Rippers sold in a day
x_2 = # of Ripper Carbons sold in a day
x_3 = # of Ripper Jrs sold in a day

Objective:	$\max 10x_1 + 30x_2 + 15x_3$	
Subject to:	$x_2 \leq 20$	A
	$x_1 + x_2 + x_3 \leq 40$	B
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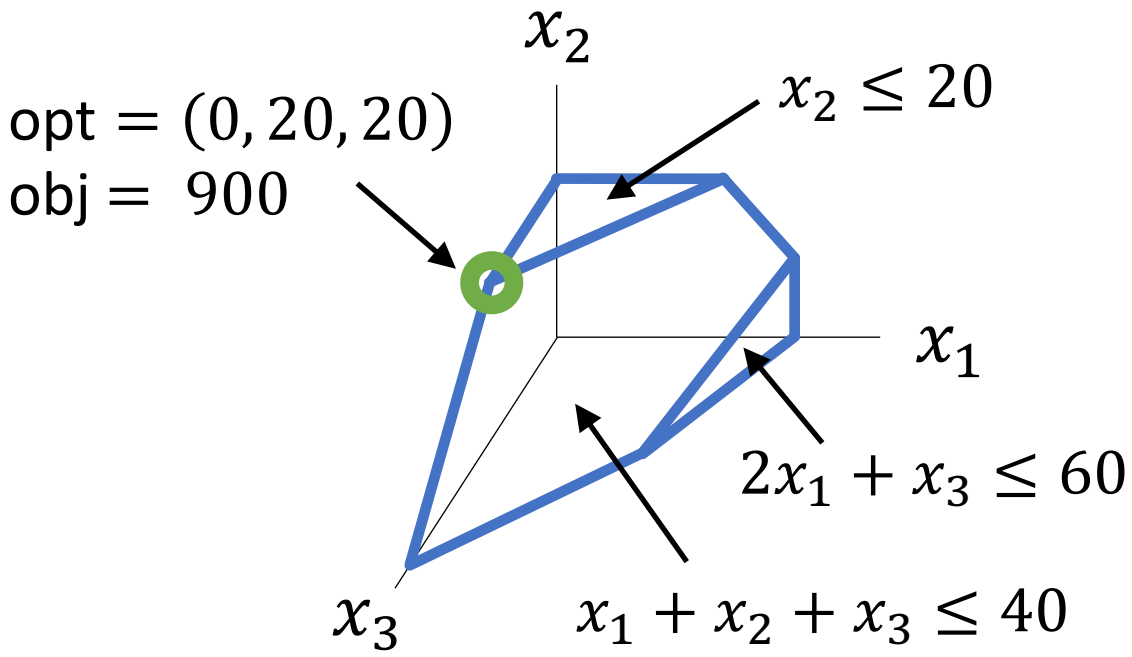
	$x_1, x_2, x_3 \geq 0$	D



?(Constraint_A) + ?(Constraint_B) + ?(Constraint_C)

Proving Optimality

Objective: $\max 10x_1 + 30x_2 + 15x_3$

Subject to:

$x_2 \leq 20$ A

$x_1 + x_2 + x_3 \leq 40$ B

$2x_1 + x_3 \leq 60$ C

Multiplier	Constraint
y_1	$x_2 \leq 20$
y_2	$x_1 + x_2 + x_3 \leq 40$
y_3	$2x_1 + x_3 \leq 60$

y_1(Constraint_A) + y_2(Constraint_B) + y_3(Constraint_C)

Proving Optimality

Objective: $\max 10x_1 + 30x_2 + 15x_3$

Subject to:

$\boldsymbol{x_2 \leq 20}$ A

$x_1 + x_2 + x_3 \leq 40$ B

$\boldsymbol{2x_1 + x_3 \leq 60}$ C

Multiplier	Constraint
y_1	$x_2 \leq 20$
y_2	$x_1 + x_2 + x_3 \leq 40$
y_3	$2x_1 + x_3 \leq 60$

y_1(Constraint_A) + y_2(Constraint_B) + y_3(Constraint_C)

$\Rightarrow \boldsymbol{y_1 x_2} + y_2 x_1 + y_2 x_2 + y_2 x_3 + \boldsymbol{2y_3 x_1 + y_3 x_3} \leq \boldsymbol{20y_1} + 40y_2 + \boldsymbol{60y_3}$

Proving Optimality

y_1(constraint_A)
$-y_2 \geq -20$

Objective: $\max 10x_1 + 30x_2 + 15x_3$

Subject to:

$x_2 \leq 20$ A

$x_1 + x_2 + x_3 \leq 40$ B

$2x_1 + x_3 \leq 60$ C

Multiplier	Constraint
y_1	$x_2 \leq 20$
y_2	$x_1 + x_2 + x_3 \leq 40$
y_3	$2x_1 + x_3 \leq 60$

y_1(Constraint_A) + y_2(Constraint_B) + y_3(Constraint_C)

$\Longrightarrow y_1x_2 + y_2x_1 + y_2x_2 + y_2x_3 + 2y_3x_1 + y_3x_3 \leq 20y_1 + 40y_2 + 60y_3$

if $y_1 = -1$, does this hold?

True or false? $y_1, y_2, y_3, \geq 0$

Proving Optimality

Objective: $\max 10x_1 + 30x_2 + 15x_3$

Subject to:

$x_2 \leq 20$ A

$x_1 + x_2 + x_3 \leq 40$ B

$2x_1 + x_3 \leq 60$ C

Multiplier	Constraint
y_1	$x_2 \leq 20$
y_2	$x_1 + x_2 + x_3 \leq 40$
y_3	$2x_1 + x_3 \leq 60$

y_1(Constraint_A) + y_2(Constraint_B) + y_3(Constraint_C)

$\Longrightarrow y_1x_2 + y_2x_1 + y_2x_2 + y_2x_3 + 2y_3x_1 + y_3x_3 \leq 20y_1 + 40y_2 + 60y_3$

y_i need to be ≥ 0, because multiplying by negative swaps the inequality sign.

Proving Optimality

Objective: $\max 10x_1 + 30x_2 + 15x_3$

Subject to:

$x_2 \leq 20$ A

$x_1 + x_2 + x_3 \leq 40$ B

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Multiplier	Constraint
y_1	$x_2 \leq 20$
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y_1(Constraint_A) + y_2(Constraint_B) + y_3(Constraint_C)

$\Longrightarrow y_1x_2 + y_2x_1 + y_2x_2 + y_2x_3 + 2y_3x_1 + y_3x_3 \leq 20y_1 + 40y_2 + 60y_3$

$\Longrightarrow (y_2 + 2y_3)x_1 + (y_1 + y_2)x_2 + (y_2 + y_3)x_3 \leq 20y_1 + 40y_2 + 60y_3$

Proving Optimality

Objective: $\max 10x_1 + 30x_2 + 15x_3$

Subject to:

$x_2 \leq 20$ A

$x_1 + x_2 + x_3 \leq 40$ B

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Multiplier	Constraint
y_1	$x_2 \leq 20$
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y_3	$2x_1 + x_3 \leq 60$

y_1(Constraint_A) + y_2(Constraint_B) + y_3(Constraint_C)

$\Rightarrow y_1x_2 + y_2x_1 + y_2x_2 + y_2x_3 + 2y_3x_1 + y_3x_3 \leq 20y_1 + 40y_2 + 60y_3$

$\Rightarrow (y_2 + 2y_3)x_1 + (y_1 + y_2)x_2 + (y_2 + y_3)x_3 \leq 20y_1 + 40y_2 + 60y_3$

≥ 10 ≥ 30 ≥ 15

Want to make **this** look like **this**.

Proving Optimality

Objective: $\max 10x_1 + 30x_2 + 15x_3$

Subject to:

$x_2 \leq 20$ A

$x_1 + x_2 + x_3 \leq 40$ B

$2x_1 + x_3 \leq 60$ C

Multiplier	Constraint
y_1	$x_2 \leq 20$
y_2	$x_1 + x_2 + x_3 \leq 40$
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y_1(Constraint_A) + y_2(Constraint_B) + y_3(Constraint_C)

$\Rightarrow y_1x_2 + y_2x_1 + y_2x_2 + y_2x_3 + 2y_3x_1 + y_3x_3 \leq 20y_1 + 40y_2 + 60y_3$

$\Rightarrow (y_2 + 2y_3)x_1 + (y_1 + y_2)x_2 + (y_2 + y_3)x_3 \leq 20y_1 + 40y_2 + 60y_3$

$\Rightarrow 10x_1 + 30x_2 + 15x_3 \leq 20y_1 + 40y_2 + 60y_3,$

If:

$y_2 + 2y_3 \geq 10$

$y_1 + y_2 \geq 30$

$y_2 + y_3 \geq 15$

$y_1, y_2, y_3 \geq 0$

Proving Optimality

Objective: $\max 10x_1 + 30x_2 + 15x_3$

Subject to:

$x_2 \leq 20$ A

$x_1 + x_2 + x_3 \leq 40$ B

$2x_1 + x_3 \leq 60$ C

Multiplier	Constraint
y_1	$x_2 \leq 20$
y_2	$x_1 + x_2 + x_3 \leq 40$
y_3	$2x_1 + x_3 \leq 60$

y_1(Constraint_A) + y_2(Constraint_B) + y_3(Constraint_C)

$\Longrightarrow y_1x_2 + y_2x_1 + y_2x_2 + y_2x_3 + 2y_3x_1 + y_3x_3 \leq 20y_1 + 40y_2 + 60y_3$

$\Longrightarrow (y_2 + 2y_3)x_1 + (y_1 + y_2)x_2 + (y_2 + y_3)x_3 \leq 20y_1 + 40y_2 + 60y_3$

$\Longrightarrow 10x_1 + 30x_2 + 15x_3 \leq 20y_1 + 40y_2 + 60y_3,$ If: $y_2 + 2y_3 \geq 10$

$y_1 + y_2 \geq 30$

$y_2 + y_3 \geq 15$

$y_1, y_2, y_3 \geq 0$

Need to find valid y_i's.

Proving Optimality

Objective: $\max 10x_1 + 30x_2 + 15x_3$

Subject to:

$x_2 \leq 20$ A

$x_1 + x_2 + x_3 \leq 40$ B

$2x_1 + x_3 \leq 60$ C

Multiplier	Constraint
y_1	$x_2 \leq 20$
y_2	$x_1 + x_2 + x_3 \leq 40$
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$y_1 + y_2 \geq 30$

$y_2 + y_3 \geq 15$

$y_1, y_2, y_3 \geq 0$

Need to find valid y_i's.

$y_1 = 10, y_2 = 20, y_3 = 10 \Longrightarrow$ objective ≤ 1600

Proving Optimality

Objective: $\max 10x_1 + 30x_2 + 15x_3$

Subject to:

$x_2 \leq 20$ A

$x_1 + x_2 + x_3 \leq 40$ B

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Multiplier	Constraint
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$y_1 + y_2 \geq 30$

$y_2 + y_3 \geq 15$

$y_1, y_2, y_3 \geq 0$

~~Need to find valid y_i's.~~

Need to find the best y_i's.

Proving Optimality

Objective: $\max 10x_1 + 30x_2 + 15x_3$

Subject to:

$x_2 \leq 20$ A

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$2x_1 + x_3 \leq 60$ C

Multiplier	Constraint
y_1	$x_2 \leq 20$
y_2	$x_1 + x_2 + x_3 \leq 40$
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y_1(Constraint_A) + y_2(Constraint_B) + y_3(Constraint_C)

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$y_2 + y_3 \geq 15$

$y_1, y_2, y_3 \geq 0$

~~Need to find valid y_i's.~~

Need to find the best y_i's.

(i.e. y_i's that make this smallest)

Proving Optimality

Objective: $\max 10x_1 + 30x_2 + 15x_3$

Subject to:

$x_2 \leq 20$ A

$x_1 + x_2 + x_3 \leq 40$ B

$2x_1 + x_3 \leq 60$ C

Multiplier	Constraint
y_1	$x_2 \leq 20$
y_2	$x_1 + x_$

y_1(Constraint A)

$+ 40y_2 + 60y_3$

$x_3)x_3 \leq 20y_1 + 40y_2 + 60y_3$

$x_2 + 150x_3 \leq \boxed{20y_1 + 40y_2 + 60y_3}$, If: $y_2 + 2y_3 \geq 10$

$y_1 + y_2 \geq 30$

$y_2 + y_3 \geq 15$

$y_1, y_2, y_3 \geq 0$

Linear Programmize™ It!

~~Need to find valid y_i's.~~

Need to find the best y_i's.

(i.e. y_i's that make this smallest)

Dual

primal

$c^T x$

s.t. $Ax \leq b$

$x \geq 0$

objective

s.t.

$b^T y$

$A^T y \geq c$

$y \geq 0$

primal

Objective: $\max 10x_1 + 30x_2 + 15x_3$

Subject to:

$x_2 \leq 20$	A
$x_1 + x_2 + x_3 \leq 40$	B
$2x_1 + x_3 \leq 60$	C

dual

Objective: $\min 20y_1 + 40y_2 + 60y_3$

Subject to:

$y_2 + 2y_3 \geq 10$

$y_1 + y_2 \geq 30$

$y_2 + y_3 \geq 15$

$y_1, y_2, y_3 \geq 0$

y_1(Constraint_A) + y_2(Constraint_B) + y_3(Constraint_C)

$\Rightarrow y_1x_2 + y_2x_1 + y_2x_2 + y_2x_3 + 2y_3x_1 + y_3x_3 \leq 20y_1 + 40y_2 + 60y_3$

$\Rightarrow (y_2 + 2y_3)x_1 + (y_1 + y_2)x_2 + (y_2 + y_3)x_3 \leq 20y_1 + 40y_2 + 60y_3$

$\Rightarrow 10x_1 + 30x_2 + 15x_3 \leq 20y_1 + 40y_2 + 60y_3,$ If:

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$y_1 + y_2 \geq 30$

$y_2 + y_3 \geq 15$

$y_1, y_2, y_3 \geq 0$

Dual

$x^* = (x_1, x_2, x_3)$

$y^* = (y_1, y_2, y_3)$

Primal

Objective: $\max c^T x$

Subject to: $A x \leq b$

$x \geq 0$

Dual

Objective: $\min b^T y$

Subject to: $A^T y \geq c$

$y \geq 0$

Objective: $\max \begin{bmatrix} 10 & 30 & 15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

Subject to: $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \leq \begin{bmatrix} 20 \\ 40 \\ 60 \end{bmatrix}$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Objective: $\min \begin{bmatrix} 20 & 40 & 60 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

Subject to: $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \geq \begin{bmatrix} 10 \\ 30 \\ 15 \end{bmatrix}$

$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Dual

Primal

Objective: $\max c^T x$

Subject to: $A x \leq b$

$x \geq 0$

Dual

Objective: $\min b^T y$

Subject to: $A^T y \geq c$

$y \geq 0$

Theorem: The dual of a dual is the original primal.

Proof: ?

Dual

Primal

Objective: $\max c^T x$

Subject to: $A x \leq b$

$x \geq 0$

Dual

Objective: $\min b^T y$

Subject to: $A^T y \geq c$

$y \geq 0$

Theorem: The dual of a dual is the original primal.

Proof:

Objective: $\min b^T y$
Subject to: $A^T y \geq c$
$y \geq 0$

→ Standard Form

Objective: $\max -b^T y$
Subject to: $-A^T y \leq -c$
$y \geq 0$

→ Dual

Objective: $\min -c^T z$
Subject to: $-A^{T^T} z \geq -b$
$z \geq 0$

→ Standard Form

Objective: $\max c^T z$
Subject to: $A z \leq b$
$z \geq 0$