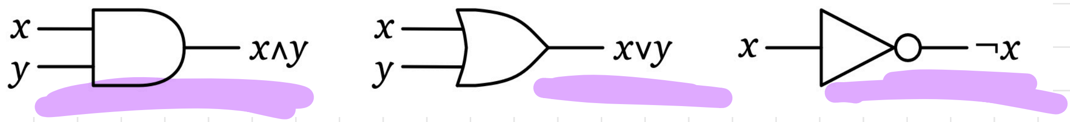
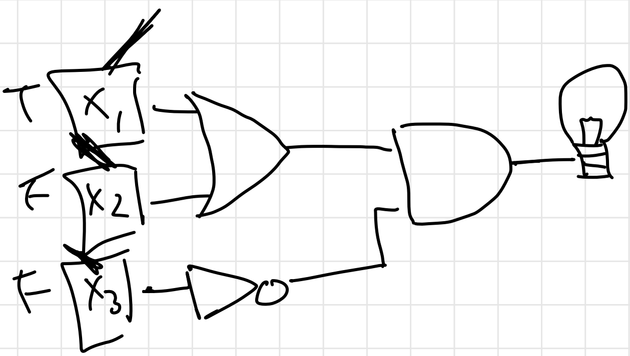
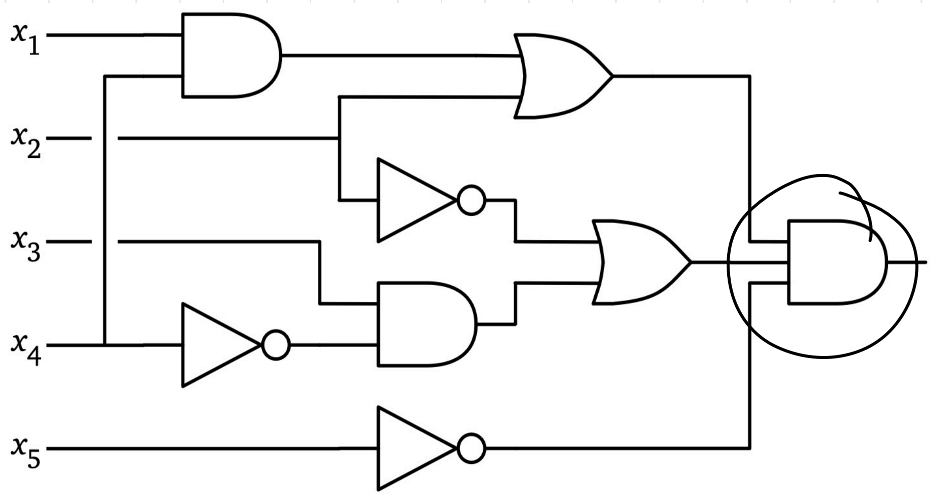




Does $x_1 = T$
$x_2 = T$ evaluate to T ?
$x_3 = T$



CIRCUITSAT: Given a boolean circuit B can we turn on the light?

satisfiable

we can verify a proposed yes answer in polynomial time.

<=>

the problem can be solved in nondeterministic polynomial time

A problem is in NP if it can be verified in poly time.

Which are in NP? why?

- ~~EVEN: given an integer >0, is it even?~~
- ~~PRIME: given an integer >0, is it prime?~~
- KVERTEXCOVER: given a graph G and an integer k, does G have a vertex cover of size k?
- SMALLESTVC: given $G = (V, E)$ and $C \subseteq V$, is C a vertex cover of smallest size for G?

NP - verify in poly time

P - solve in poly time

$X \in P \Rightarrow X \in NP$ yes

$X \in NP \Rightarrow X \in P$

$P = NP$ or $P \neq NP$?

Cook-Levin: Circuit SAT $\in P \Rightarrow P = NP$

X is NP-hard: there is a polynomial time reduction from SAT to X

X is NP-hard: there is a poly time reduction from some ~~known~~ known NP-hard problem to X.

3SAT : Given a boolean formula in 3CNF, is it satisfiable?

$$((x_1 \wedge x_2) \vee x_3) \vee \overline{x_3})$$

$$
\begin{aligned}
&(y_1 \vee \overline{x_1} \vee \overline{x_4}) \wedge (\overline{y_1} \vee x_1 \vee z_1) \wedge (\overline{y_1} \vee x_1 \vee \overline{z_1}) \wedge (\overline{y_1} \vee x_4 \vee z_2) \wedge (\overline{y_1} \vee x_4 \vee \overline{z_2}) \\
&\wedge (y_2 \vee x_4 \vee z_3) \wedge (y_2 \vee x_4 \vee \overline{z_3}) \wedge (\overline{y_2} \vee \overline{x_4} \vee z_4) \wedge (\overline{y_2} \vee \overline{x_4} \vee \overline{z_4}) \\
&\wedge (y_3 \vee \overline{x_3} \vee \overline{y_2}) \wedge (\overline{y_3} \vee x_3 \vee z_5) \wedge (\overline{y_3} \vee x_3 \vee \overline{z_5}) \wedge (\overline{y_3} \vee y_2 \vee z_6) \wedge (\overline{y_3} \vee y_2 \vee \overline{z_6}) \\
&\wedge (\overline{y_4} \vee y_1 \vee x_2) \wedge (y_4 \vee \overline{x_2} \vee z_7) \wedge (y_4 \vee \overline{x_2} \vee \overline{z_7}) \wedge (y_4 \vee \overline{y_1} \vee z_8) \wedge (y_4 \vee \overline{y_1} \vee \overline{z_8}) \\
&\wedge (y_5 \vee x_2 \vee z_9) \wedge (y_5 \vee x_2 \vee \overline{z_9}) \wedge (\overline{y_5} \vee \overline{x_2} \vee z_{10}) \wedge (\overline{y_5} \vee \overline{x_2} \vee \overline{z_{10}}) \\
&\wedge (y_6 \vee x_5 \vee z_{11}) \wedge (y_6 \vee x_5 \vee \overline{z_{11}}) \wedge (\overline{y_6} \vee \overline{x_5} \vee z_{12}) \wedge (\overline{y_6} \vee \overline{x_5} \vee \overline{z_{12}}) \\
&\wedge (\overline{y_7} \vee y_3 \vee y_5) \wedge (y_7 \vee \overline{y_3} \vee z_{13}) \wedge (y_7 \vee \overline{y_3} \vee \overline{z_{13}}) \wedge (y_7 \vee \overline{y_5} \vee z_{14}) \wedge (y_7 \vee \overline{y_5} \vee \overline{z_{14}}) \\
&\wedge (y_8 \vee \overline{y_4} \vee \overline{y_7}) \wedge (\overline{y_8} \vee y_4 \vee z_{15}) \wedge (\overline{y_8} \vee y_4 \vee \overline{z_{15}}) \wedge (\overline{y_8} \vee y_7 \vee z_{16}) \wedge (\overline{y_8} \vee y_7 \vee \overline{z_{16}}) \\
&\wedge (y_9 \vee \overline{y_8} \vee \overline{y_6}) \wedge (\overline{y_9} \vee y_8 \vee z_{17}) \wedge (\overline{y_9} \vee y_6 \vee z_{18}) \wedge (\overline{y_9} \vee y_6 \vee \overline{z_{18}}) \wedge (\overline{y_9} \vee y_8 \vee \overline{z_{17}}) \\
&\wedge (y_9 \vee z_{19} \vee z_{20}) \wedge \underbrace{(y_9 \vee \overline{z_{19}} \vee z_{20})}_{\text{clause}} \wedge (y_9 \vee z_{19} \vee \overline{z_{20}}) \wedge (y_9 \vee \overline{z_{19}} \vee \overline{z_{20}})
\end{aligned}
$$

and

T T T

ex: $(a \vee b \vee \bar{c}) \wedge (b \vee \bar{d} \vee e) \wedge (\bar{b} \vee e \vee \bar{e})$

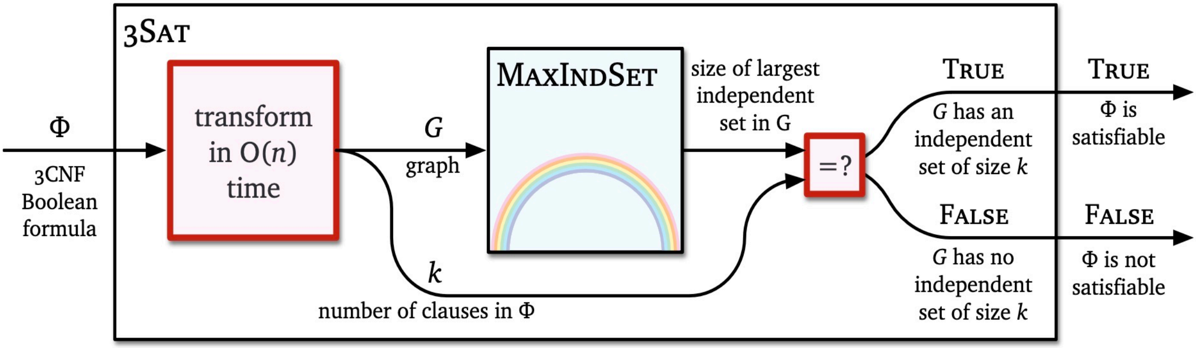
is it satisfiable? $a, b, \bar{c}$

SAT reduces to 3SAT in polytime

$\Rightarrow$ 3SAT is NP-hard

Claim: Max Independent Set is NP hard.

Proof: we give a polytime reduction from 3SAT to Max Ind Set.



3SAT(Φ):

Transform Φ to G

K = # of clauses in Φ

n = MaxIndSet(G)

if n = K:

return T

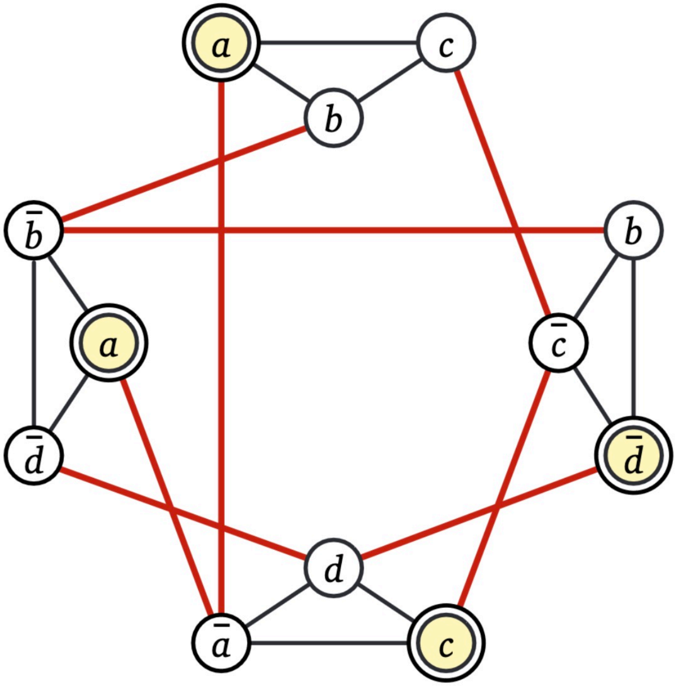
else

return F

Given

$(a \vee b \vee c) \wedge (b \vee \bar{c} \vee \bar{d}) \wedge (\bar{a} \vee c \vee d) \wedge (a \vee \bar{b} \vee \bar{d})$

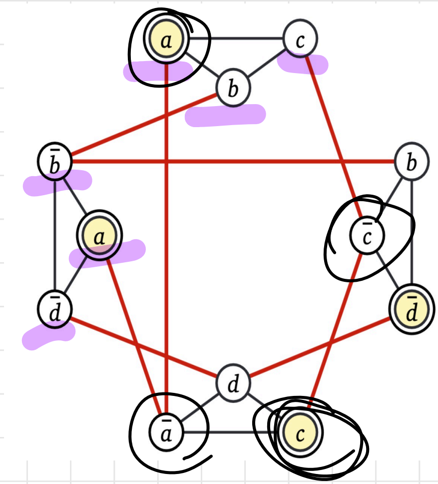
transform to G



$(\boxed{a} \lor b \lor c) \land (b \lor \bar{c} \lor \boxed{\bar{d}}) \land (\bar{a} \lor c \lor d) \land (a \lor \bar{b} \lor \bar{d})$

Transform (Φ):

- make 1 triangle per clause
- connect inverses



Claim:

Φ is satisfiable

$\Leftrightarrow$

G has an indep. set of size k

Proof:

$\Rightarrow$

Φ is satisfiable.

Fix a satisfying assn.

Each clause has a T literal.

Select one from each and add to a set S.

S is an IS.