CSCI 432/532

Advanced Algorithms Topics

<u>course goals</u>

- what is computation?
- what is computable?
- what is computable <u>quickly</u>?
- what strategies can we use to design algorithms to compute quickly?

- practice strategies for understanding difficult / non-intuitive ideas
- practice communicating about CS theory, algorithms
  - in writing
  - in presentation

Today: strings — practice recursion, induction

# Definitions

A string is either

- nothing
- a symbol followed by a string

  ↳ from alphabet $\Sigma$, some non-empty set

A string is either

- $\varepsilon$ (empty string)
- $(a, x)$ where $a \in \Sigma$ and $x$ is a string

$$\text{string} = (s, \underline{\text{tring}})$$
$$= (s, (t, \text{ring}))$$
$$\vdots$$
$$= (s, (t, (r, (i, (n, (g))))))$$

but we will omit ( ,

$\Sigma^* =$ set of all strings over $\Sigma$

ex string $\in \boxed{\{a, b, c, \ldots, x, y, z\}^*}$

$0110 \in \{0,1\}^*$

$attgca \in \{a,c,t,g\}^*$

**length function** – "# of symbols"

let $w$ be a string

$$|w| := \begin{cases} 0 & \text{if } w = \varepsilon \\ 1 + |x| & \text{if } w = ax \text{ for } a \in \Sigma, \text{ string } x \end{cases}$$

**ex** $|\text{string}| = 1 + |\text{tring}|$

$= 1 + 1 + |\text{ring}|$

$\vdots$

$= 1+1+1+1+1+1+|\varepsilon|$

$= 6 + 0 = 6$

**concatenation** – "paste one after another"

$w, z$ strings

$$w \cdot z := \begin{cases} z & \text{if } w = \varepsilon \\ a(x \cdot z) & \text{if } w = ax \end{cases}$$

**ex** $\text{foot} \cdot \text{ball} = f(\text{oot} \cdot \text{ball})$

$= f(o(\text{ot} \cdot \text{ball}))$

$= f(o(o(t \cdot \text{ball})))$

$= f(o(o(t(\varepsilon \cdot \text{ball}))))$

$= f(o(o(t(\text{ball}))))$

$= \text{football}$

**Theorem** for any string $w$, $w \cdot \varepsilon = w$.

Let $w$ be an arbitrary string.

Assume that for all strings $x$ that are shorter than $w$, $x \cdot \varepsilon = x$.

There are two cases:

Case 1: $w = \varepsilon$.

| | |
|---|---|
| $w \cdot \varepsilon = \varepsilon \cdot \varepsilon$ | because $w = \varepsilon$ |
| $= \varepsilon$ | def of $\cdot$ |
| $= w$ | because $w = \varepsilon$ |

Case 2: $w = ay$

| | |
|---|---|
| $w \cdot \varepsilon = (ay) \cdot \varepsilon$ | because $w = ay$ |
| $= a(y \cdot \varepsilon)$ | def. of $\cdot$ |
| $= ay$ | by inductive hypothesis |
| $= w$ | because $w = ay$ |

in both cases

Therefore, $w \circ \varepsilon = w$.

<u>Theorem</u> for all strings $w$ and $z$,

$|w \circ z| = |w| + |z|$.

<u>Proof</u> Let $w, z$ be arbitrary strings.

Assume for all $x$ shorter than $w$,

$|x \circ z| = |x| + |z|$

case 1: $w = \varepsilon$

| | | |
|---|---|---|
| $\|w \circ z\|$ | $= \|\varepsilon \circ z\|$ | because $w = \varepsilon$ |
| | $= \|z\|$ | def of $\circ$ |
| | $= 0 + \|z\|$ | math |
| | $= \|\varepsilon\| + \|z\|$ | def of $\|\|$ |
| | $= \|w\| + \|z\|$ | because $w = \varepsilon$ |

case 2: $w = ax$

| | | |
|---|---|---|
| $\|w \circ z\|$ | $= \|(ax) \circ z\|$ | because $w = ax$ |
| | $= \|a(x \circ z)\|$ | def. of $\circ$ |
| | $= 1 + \|x \circ z\|$ | def of $\|\|$ |
| | $= 1 + \|x\| + \|z\|$ | IH |
| | $= \|ax\| + \|z\|$ | def of $\|\|$ |
| | $= \|w\| + \|z\|$ | because $w = ax$ |

Therefore, $|w \circ z| = |w| + |z|$.