

CSCI 432/532, Spring 2024
Problem Set 9

i. Let M be an arbitrary Turing machine. Prove the following claims.

(a) There is a Turing machine M^R such that

$$\text{ACCEPT}(M^R) = \text{REJECT}(M)$$

and

$$\text{REJECT}(M^R) = \text{ACCEPT}(M).$$

(b) There is a Turing machine M^A such that

$$\text{ACCEPT}(M^A) = \text{ACCEPT}(M)$$

and

$$\text{REJECT}(M^A) = \emptyset.$$

(c) There is a Turing machine M^H such that

$$\text{ACCEPT}(M^H) = \text{HALT}(M)$$

and

$$\text{REJECT}(M^H) = \emptyset.$$

2. Let $\text{SELFACCEPT} = \{\langle M \rangle : M \text{ accepts } \langle M \rangle\}$. Prove that SELFACCEPT is undecidable.

3. Let $\text{ACCEPT} = \{\langle M, w \rangle : M \text{ accepts } w\}$. Prove that ACCEPT is undecidable.

To think about later:

i. Prove that each of the following languages is undecidable.

(a) $\text{ALWAYACCEPT} = \{\langle M \rangle : \text{ACCEPT}(M) = \Sigma^*\}$

(b) $\text{ALWAYREJECT} = \{\langle M \rangle : \text{REJECT}(M) = \Sigma^*\}$

(c) $\text{ALWAYHALT} = \{\langle M \rangle : \text{HALT}(M) = \Sigma^*\}$

(d) $\text{ALWAYDIVERGE} = \{\langle M \rangle : \text{DIVERGE}(M) = \Sigma^*\}$