CSCI 432/532, Spring 2025
Homework 4
Due Tuesday, February 18, 2025 at 11:59pm Mountain Time

Submission Requirements

- Type or clearly hand-write your solutions into a PDF format so that they are legible and professional. Submit your PDF to the appropriate Canvas dropbox.
- Do not submit your first draft. Type or clearly re-write your solutions for your final submission.
- You may work with a group of up to three students and submit ***one single document*** for the group. Just be sure to list all group members at the top of the document.
- When possible, the homework will include at least one fully solved problem, similar to that week's assigned problems, together with the rubric we would use to grade this problem if it appeared in an actual homework or exam. These model solutions show our recommendations for structure, presentation, and level of detail in your homework solutions. (Obviously, the actual *content* of your solutions won't match the model solutions, because your problems are different!)

Academic Integrity

Remember, you may access ***any*** resource in preparing your solution to the homework. However, you must

- write your solutions in your own words, and
- credit every resource you use (for example: "Bob Smith helped me on problem 2. He took this course at UM in Fall 2020"; "I found a solution to a problem similar to this one in the lecture notes for a different course, found at this link: www.profzeno.com/agreatclass/lecture10"; "I asked ChatGPT how to solve problem 1 part (c); "I put my solution for problem 1 part (c) into ChatGPT to check that it was correct and it caught a missing case and suggested some grammer fixes.") If you use the provided LaTeX template, you can use the `sources` environment for this. Ask if you need help!

Grading Rubrics

For the context-free grammar problems:

Context-free grammar rubric. 10 points =

+ 2 for a syntactically correct context-free grammar.
+ 4 for a *brief* English explanation of your context-free grammar. This is how you argue that your CFG is correct. We do not want a *transcription*; don't just translate the CFG *notation* into English.
  - A CFG without an explanation cannot receive the 4 points for correctness, so its maximum

score is 2.

+ 4 for correctness

For the Turing machine description:

TM rubric. 10 points =

+ 2 for an English description of a Turing machine, regardless of correctness.
+ 6 for correctness.
+ 2 for an appropriate mix of detail and high-level intuition.

For the Turing machine simulation problem:

TM simulation rubric. 10 points =

+ 2 for an English description of how to simulate a standard Turing machine with the new type of Turing machine or how to prove that a standard Turing machine can't be simulated with the new type of Turing machine.
+ 6 for correctness.
+ 2 for an appropriate mix of detail and high-level intuition.

1. Give context-free grammars for the following languages, and clearly explain how they work and the set of strings generated by each nonterminal. Be sure to reference the rubric–note that a CFG without an explanation may receive little or no credit. On the other hand, we do not need a formal proof of correctness.

   (a) $\{0^a 10^b 10^c : b = 2a + 2c\}$.

   (b) The set of all palindromes in Σ^* whose lengths are divisible by 7.

2. Describe (as in the problem session solutions—note that you do *not* need to give a full specification of states, transitions, etc.) a Turing machine that decides the following language: $\{ww : w \in \{0,1\}^*\}$.

3. A ***Turing machine with stay put instead of left*** is similar to an ordinary Turing machine, but the transition function has the form $\delta : Q \times \Gamma \to Q \times \Gamma \times \{R, S\}$. At each point, the machine can move its head right or let it stay put in the same position.

   (a) Show that this Turing machine variant is *not* equivalent to a standard Turing machine.

   (b) What class of languages do these machines recognize?

Solved problems

Give context-free grammars for the following languages over the alphabet $\Sigma = \{0, 1\}$. Clearly explain how they work and the set of strings generated by each nonterminal. Grammars with unclear or missing explanations may receive little or no credit; on the other hand, we do *not* want formal proofs of correctness.

1. In any string, a ***run*** is a maximal non-empty substring of identical symbols. For example, the string $0111000011001 = 0^1 1^3 0^4 1^2 0^2 1^1$ consists of six runs.

   Let L_a be the set of all strings in Σ^* that contain two runs of 0s of equal length. For example, L_a contains the strings 01101111 and 01001011100010 (because each of those strings contains more than one run of 0s of length 1) but L_a does not contain the strings 000110011011 and 00000000111.

   Solution:

$S \to ACB$	strings with two blocks of 0s of same length
$A \to \varepsilon \mid X1$	empty or ends with 1
$B \to \varepsilon \mid 1X$	empty or starts with 1
$C \to 0C0 \mid 0D0$	$0^n y 0^n$, where y starts and ends with 1
$D \to 1 \mid 1X1$	starts and ends with 1
$X \to \varepsilon \mid 1X \mid 0X$	all strings: $(0+1)^*$

   Every string in L has the form $x0^n y0^n z$, where x is either empty or ends with 1, y starts and ends with 1, and z is either empty or begins with 1. Nonterminal A generates the prefix x; non-terminal B generates the suffix z; nonterminal C generates the matching runs of 0s, and nonterminal D generates the interior string y.

The same decomposition can be expressed more compactly as follows:

$S \to B \mid B1A \mid A1B \mid A1B1A$	strings with two blocks of 0s of same length
$A \to 1A \mid 0A \mid \varepsilon$	all strings: $(0+1)^*$
$B \to 0B0 \mid 010 \mid 01A10$	$0^n y 0^n$, where y starts and ends with 1

■

Rubric: 5 points = 3 for clearly correct grammar + 2 for clear explanation. These are not the only correct solutions.

2. $L_b = \{w \in \Sigma^* \mid w \text{ is } \textit{not} \text{ a palindrome}\}$.

Solution:

$S \to 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid A$	non-palindromes
$A \to 0B1 \mid 1B0$	start and end with different symbols
$B \to 0B \mid 1B \mid \varepsilon$	all strings

Every non-palindrome w can be decomposed as either $w = x0y1z$ or $w = x1y0z$, for some substrings x, y, z such that $|x| = |z|$. Non-terminal S generates the prefix x and matching-length suffix z; non-terminal A generates the distinct symbols, and non-terminal B generates the interior substring y. ■

Solution:

$S \to 0S0 \mid 1S1 \mid A$	non-palindromes
$A \to 0B1 \mid 1B0$	start and end with different symbols
$B \to 0B \mid 1B \mid \varepsilon$	all strings

Every non-palindrome w must have a prefix x and a substring y such that either $w = x0y1x^R$ or $w = x1y0x^R$. Specifically, x is the longest common prefix of w and w^R. In the first case, the grammar generates w as follows:

$$S \rightsquigarrow^* x\,A\,x^R \rightsquigarrow x\,0B1\,x^R \rightsquigarrow^* x\,0y1\,x^R = w$$

The derivation for $w = x1y0x^R$ is similar. ■

Rubric: 5 points = 3 for clearly correct grammar + 2 for clear explanation. These are not the only correct solutions.