CSCI 432/532, Spring 2025

Homework 8

Due Monday, March 25, 2025 at 11:59pm Mountain Time

Submission Requirements

- Type or clearly hand-write your solutions into a PDF format so that they are legible and professional. Submit your PDF to the appropriate Canvas dropbox.
- Do not submit your first draft. Type or clearly re-write your solutions for your final submission.
- You may work with a group of up to three students and submit ***one single document*** for the group. Just be sure to list all group members at the top of the document.
- When possible, the homework will include at least one fully solved problem, similar to that week's assigned problems, together with the rubric we would use to grade this problem if it appeared in an actual homework or exam. These model solutions show our recommendations for structure, presentation, and level of detail in your homework solutions. (Obviously, the actual *content* of your solutions won't match the model solutions, because your problems are different!)

Academic Integrity

Remember, you may access ***any*** resource in preparing your solution to the homework. However, you must

- write your solutions in your own words, and
- credit every resource you use (for example: "Bob Smith helped me on problem 2. He took this course at UM in Fall 2020"; "I found a solution to a problem similar to this one in the lecture notes for a different course, found at this link: www.profzeno.com/agreatclass/lecture10"; "I asked ChatGPT how to solve problem 1 part (c); "I put my solution for problem 1 part (c) into ChatGPT to check that it was correct and it caught a missing case and suggested some grammer fixes.") If you use the provided LaTeX template, you can use the `sources` environment for this. Ask if you need help!

1. All that "residual graph" stuff seems like too much work. Your friend proposes the following, simpler algorithm to compute a max flow:

```
MaxFlow(G, c, s, t):
  For every edge e in G:
      f(e) ← 0
  While there is a path from s to t:
      π ← an arbitrary path from s to t
      F ← minimum capacity of any edge in π
      For every edge in π:
          f(e) ← f(e) + F
          If c(e) = F:
              Remove e from G
          Else:
              c(e) ← c(e) − F
  Return f
```

   (a) Give a graph where MaxFlow fails to compute the maximum flow. Give the output flow and the sequence of paths chosen by the algorithm. (8 points)

   (b) How did you go about constructing your example graph? (There is no right answer here. I'm just curious.) (2 points)

2. In order to match computer science students to summer internships, the University of Montana computer science department has each student list the companies that they are willing to work for and each company list the students that they are willing to hire. Given these lists, you would like to find an assignment of students to companies such that the maximum number of students are given internships.

   Give a reduction to the maximum flow problem to solve this problem. (10 points)

3. An important task for wildlife managers is to control the spread of invasive species. Given a graph where nodes represent habitats and edges represent migration corridors between nodes, give an algorithm (yes, using maximum flow) to find the minimum number of habitats (not migration corridors!) that you need to make inhospitable for the species to prevent it from spreading from habitat s to habitat t. (Assume that the invasive species is already so established in s that it cannot be controlled, and habitat t is too delicate to have the procedure applied.) (10 points)