We can define languages using regular expressions...

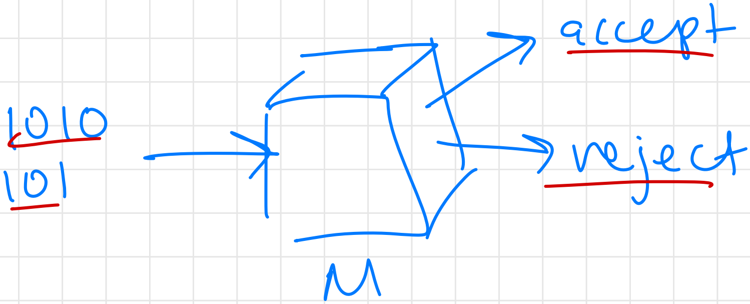
sets of strings

ex $((0+1)(0+1))^*$

even length binary strings

1 X

Another way to define languages:

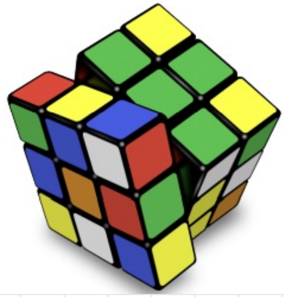


$L(M)$ = set of all strings accepted by M

"language of M"

M recognizes language L

Finite State Machines



want to design a finite state machine to recognize

$\{ w \in \{0,1\}^* : |w| \bmod 5 = 0 \}$

↳ length of w is divisible by 5

0110101011

011

idea 1:
keep track of $|w|$

✗ not finite state

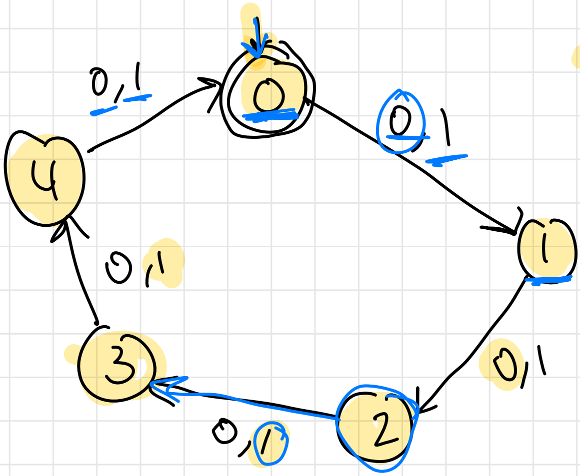
as an algorithm:

```
rem = 0
for i = 1 to |w|:
    rem = (rem+1) mod 5
return (rem == 0)
```

idea 2:
keep track of remainder ✓

0, 1, 2, 3, 4

as a finite state machine



1011

picture / graph

Deterministic Finite Automaton (DFA)

$(Q, s, \Sigma, A, \delta)$

Q : set of states $\{0,1,2,3,4\}$

$s \in Q$: start state 0

Σ : input alphabet $\{0,1\}$

$A \subseteq Q$: set of accepting states $\{0\}$

$\delta : Q \times \Sigma \to Q$ transition function

delta — takes in (state, symbol) — returns state

$\delta(2,1) = 3$

δ : table

	0	1
0	1	1
1	2	2
2	3	3
3	4	4
4	0	0

symbol

$\delta(q, a) = (q+1) \bmod 5$

state

math notation

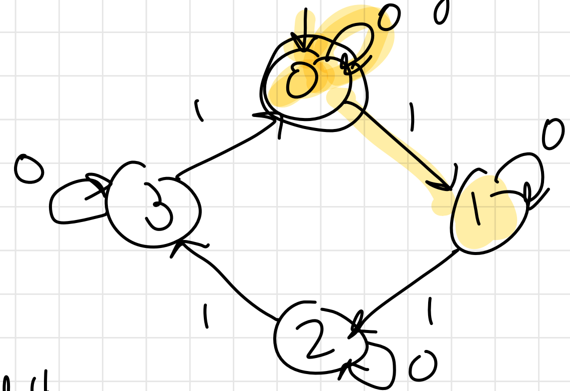
Make a DFA that recognizes binary strings where # 1s is divisible by 4.

$Q = \{0, 1, 2, 3\}$

$s = 0$

$A = \{0\}$

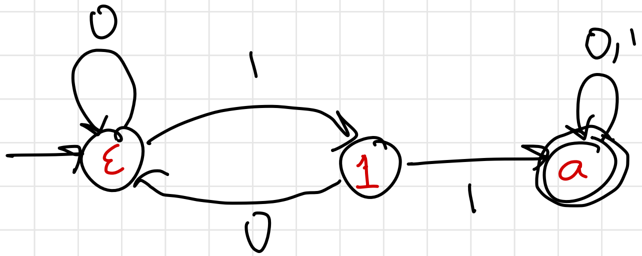
$\delta(q, a) = (q + a) \bmod 4$



0010

with table:

what strings does this machine accept?



accepts all strings w/ 11 as a substring

what do the states mean?

a = accept - we've seen two 1s at some point ~~before~~

1 = seen one 1 so far but haven't seen 11

ε = just saw 0 or start of string haven't seen 11 yet

Give a DFA that recognizes binary strings divisible by 5.

8 4 2 1
1010 is 10 in decimal accept

16 8 4 2 1
00111 is 7 in decimal reject

i	$w[1..i]$	*value*	*rem*
0	ε		
1	0		
2	00		
3	001		
4	0010		
5	00101		
6	001011		
7	0010111		
8	00101110		
9	001011101		
10	0010111011		
11	00101110110		