Maximizing Profit

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2. MFC can sell up to 30 Rippers and 20 Ripper Carbons per day.
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Day's Profit: ?

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Day's Profit: $10x_1 + 30x_2$

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Objective: $\max 10x_1 + 30x_2$

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Objective: $\max 10x_1 + 30x_2$

Subject to: ?

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Let, $x_1 =$ # of Rippers sold in a day, $x_2 =$ # of Ripper Carbons sold in a day.

Objective: $\max 10x_1 + 30x_2$

Subject to: $x_1 \leq 30$

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Subject to: $x_1 \leq 30$

$x_2 \leq 20$

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Subject to:
$$x_1 \leq 30$$
$$x_2 \leq 20$$
$$x_1 + x_2 \leq 40$$

Linear Program (LP)

x_1 = # of Rippers sold
x_2 = # of Ripper Carbons

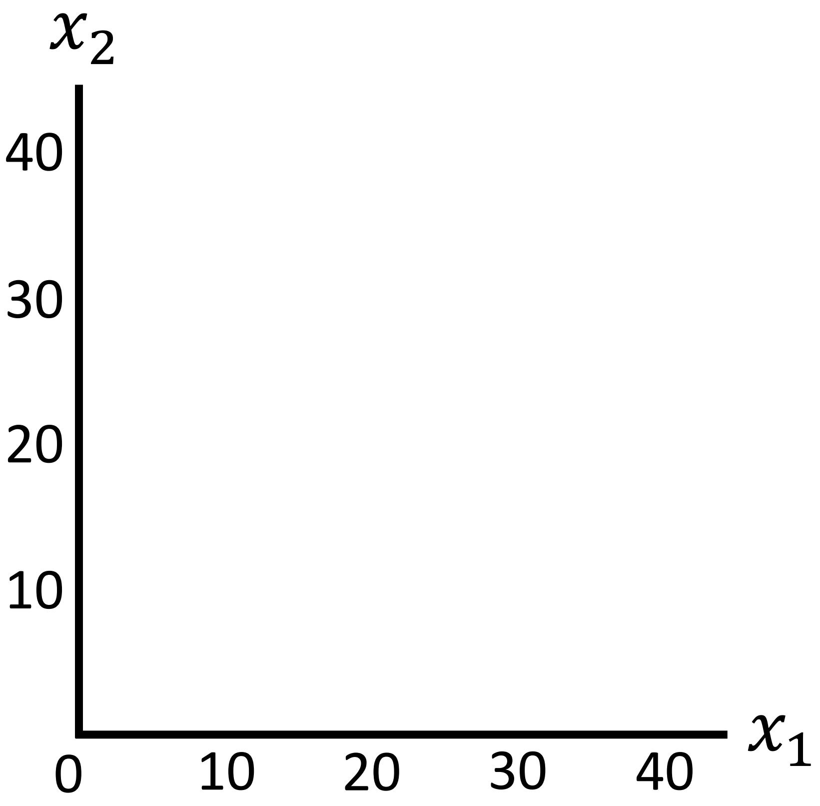
Objective: $\max 10x_1 + 30x_2$
Subject to: $x_1 \leq 30$
$x_2 \leq 20$
$x_1 + x_2 \leq 40$
$x_1, x_2 \geq 0$

variables
real #s — poly time solvable
if integer - not poly time solvable
unless P=NP

objective function
- can be min or max
- linear combination of vars
~~max $x_1 x_2$ +~~ x_1^2 quadratic programming

constraints
- linear combs. of variables
- can be $\leq, \geq, =$

Maximizing Profit



$x_1 =$ # of Rippers sold in a day
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Objective: $\max 10x_1 + 30x_2$

Subject to:
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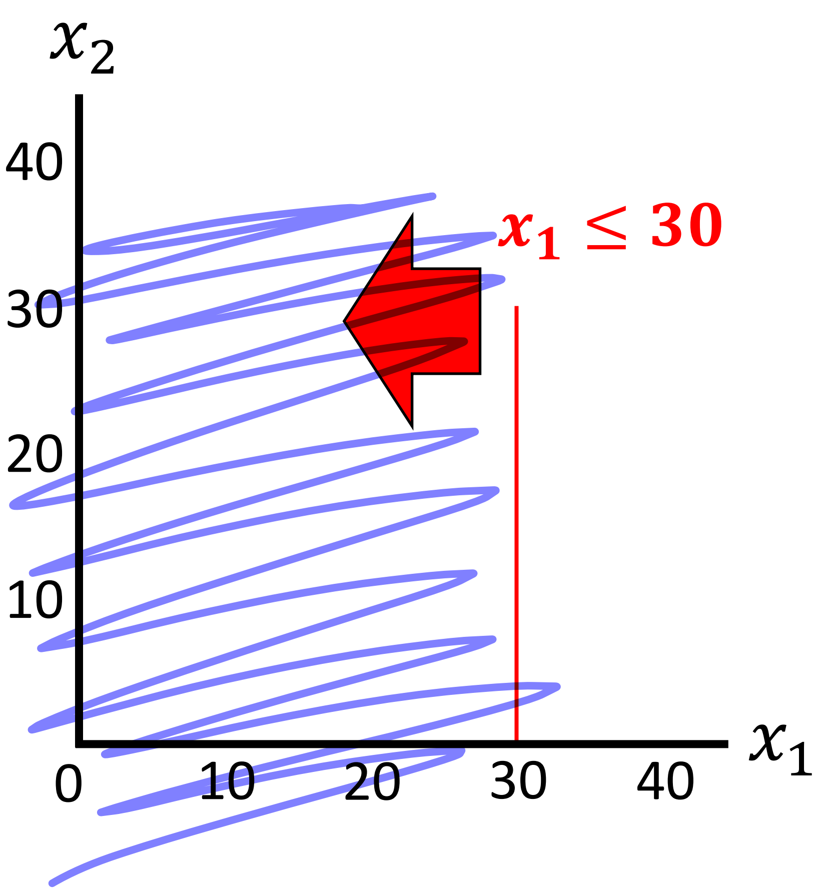

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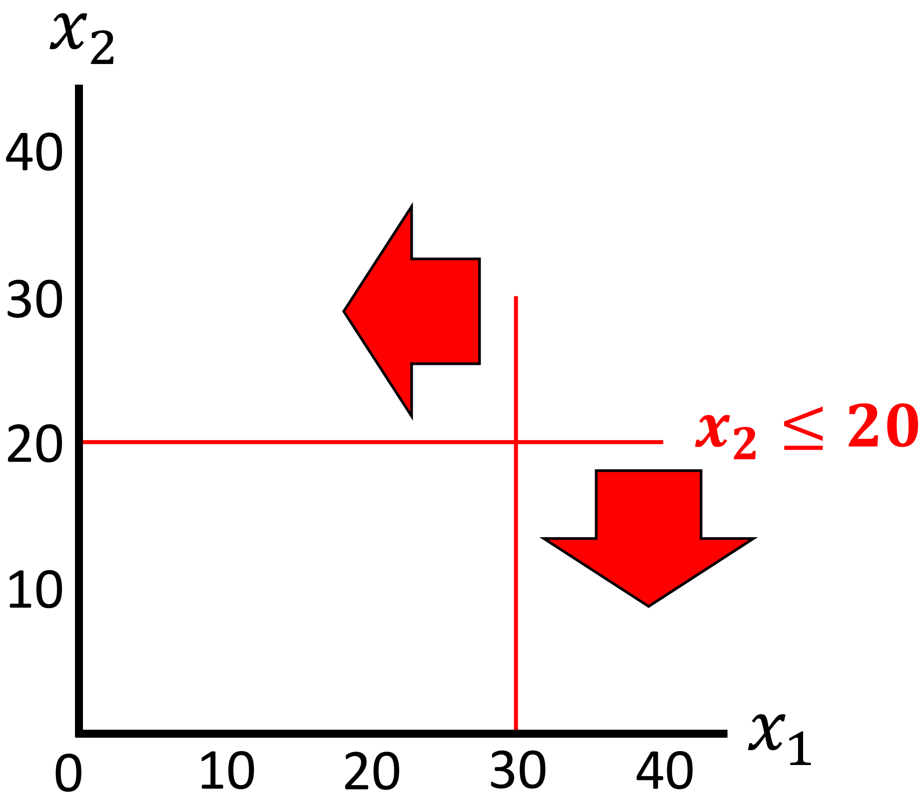

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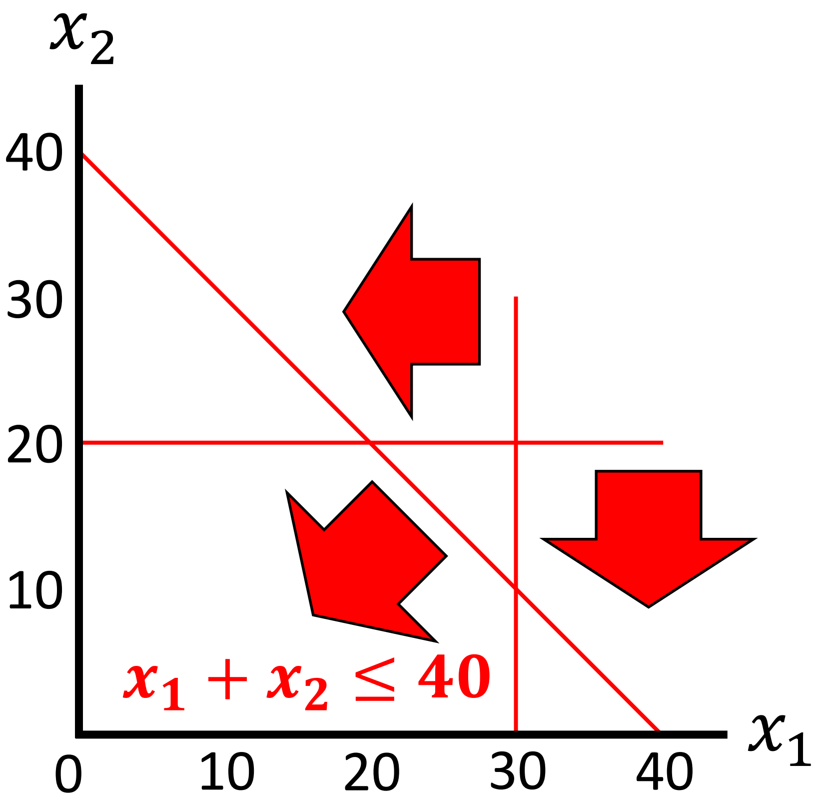

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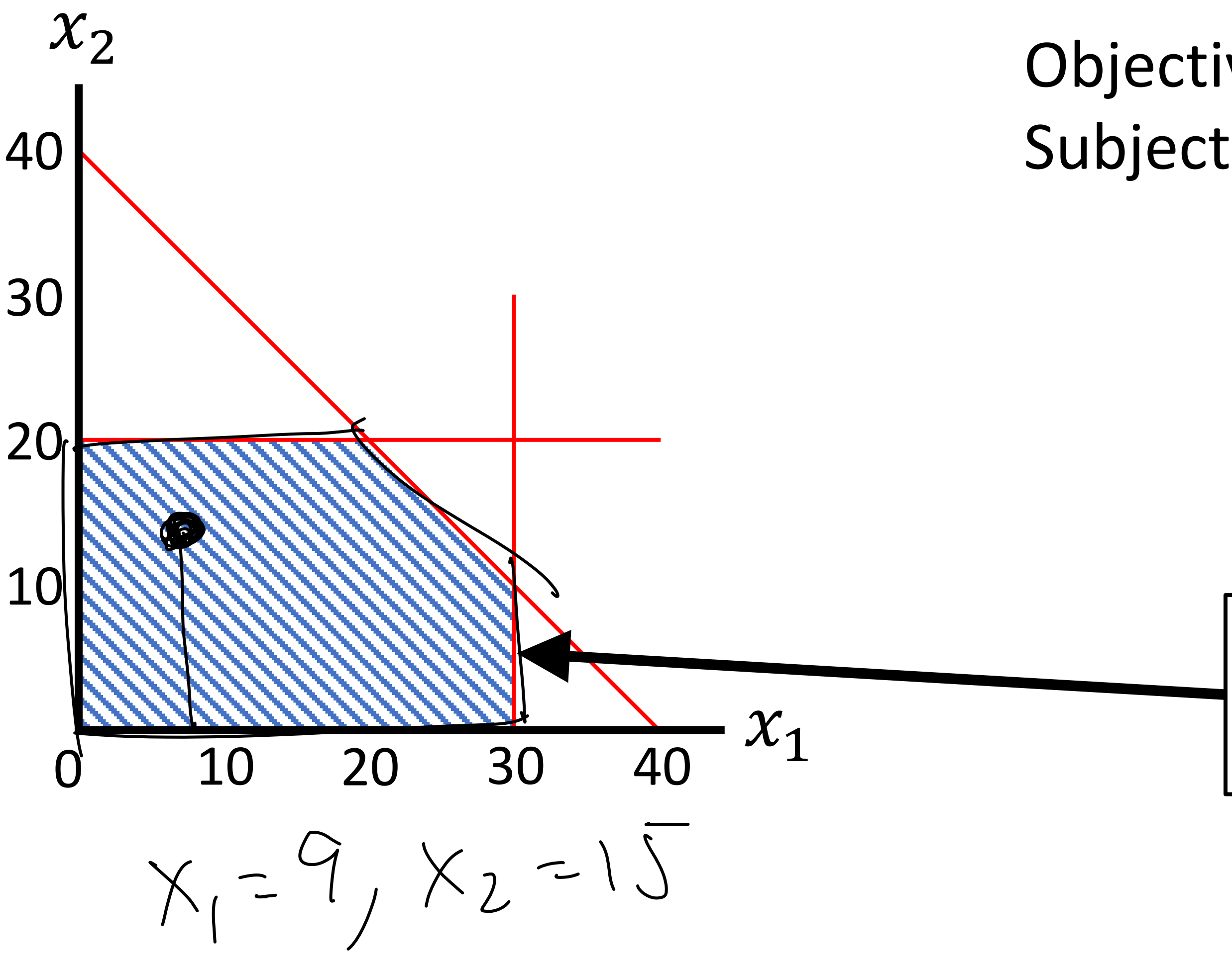
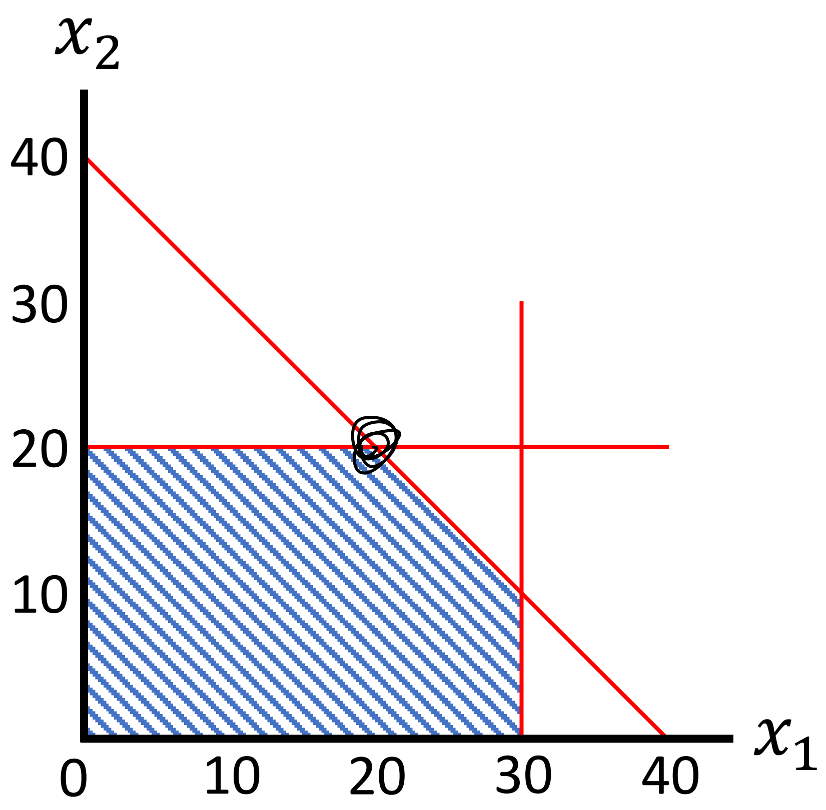


Feasible Region
(area where *all* valid solutions lie)

Maximizing Profit



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Objective: $\max 10x_1 + 30x_2$
Subject to: $x_1 \leq 30$
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$x_1, x_2 \geq 0$

What is the optimal value?

Maximizing Profit

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Objective: $\max 10x_1 + 30x_2$
Subject to: $x_1 \leq 30$
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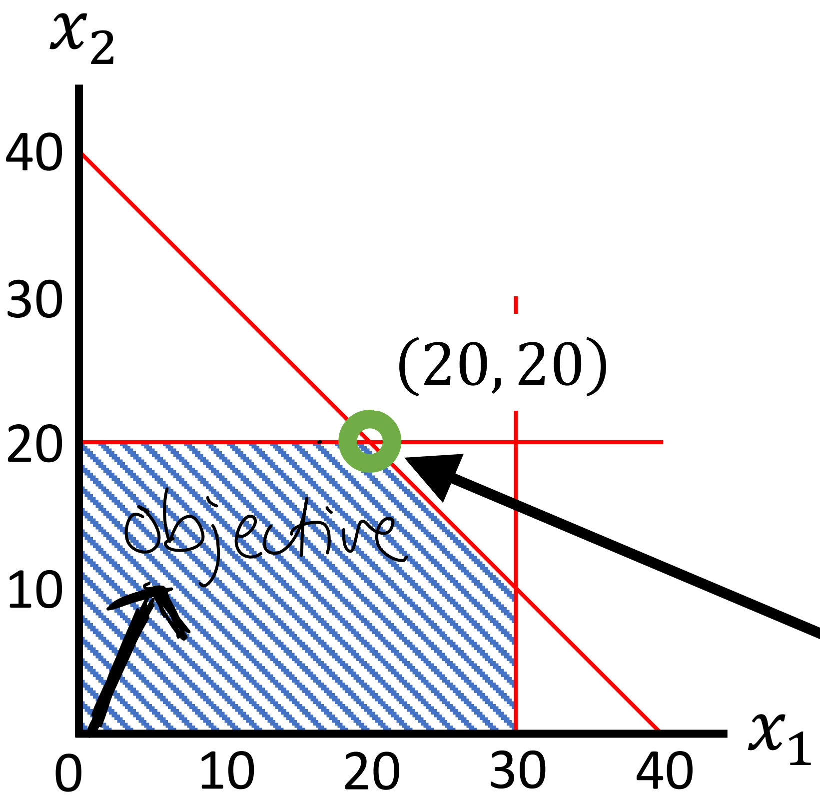

obj = $10\times20 + 30\times20 = 800$

Maximizing Profit Modification

MT Frisbee Company (MFC) wants to introduce a third frisbee aimed at kids: Ripper Jr. The Jr yields a profit of \$15. Unfortunately, the Ripper and Ripper Jr use the same machine (two hours per frisbee for the Ripper and one hour for the Ripper Jr). There are only 60 machine hours available each day.

x_1 = # of Rippers sold in a day
x_2 = # of Ripper Carbons sold in a day

x_3 = # ripper jrs sold /day

Objective: $\max 10x_1 + 30x_2 + 15x_3$

Subject to:
$x_1 \leq 30$
$x_2 \leq 20$
$x_1 + x_2 \leq 40$
$2x_1 + x_3 \leq 60$

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Subject to: $x_1 \leq 30$
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New formulation?

Maximizing Profit Modification

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Objective: $\max 10x_1 + 30x_2 + 15x_3$

Subject to:
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Subject to: $x_2 \leq 20$
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What does the feasible region look like now?

Maximizing Profit Modification

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Subject to:
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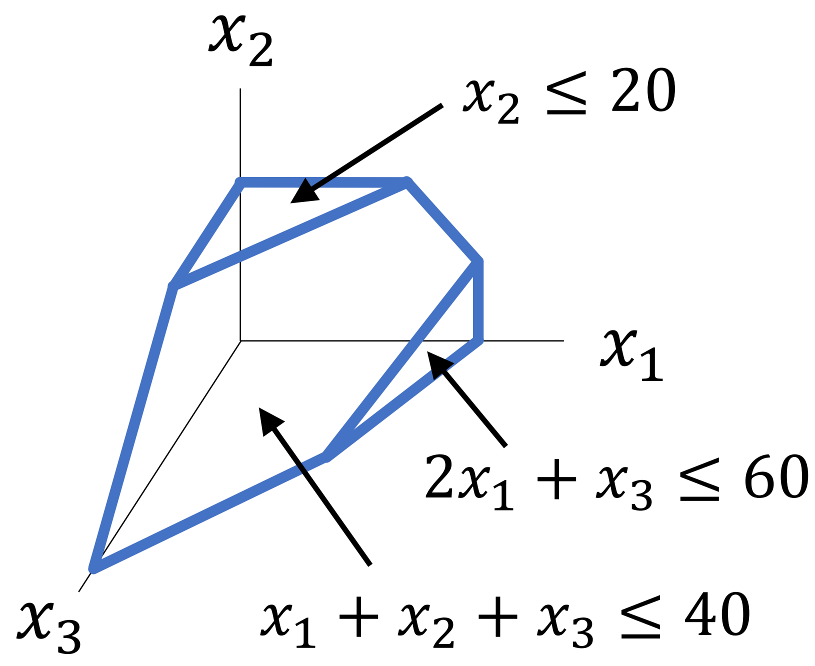

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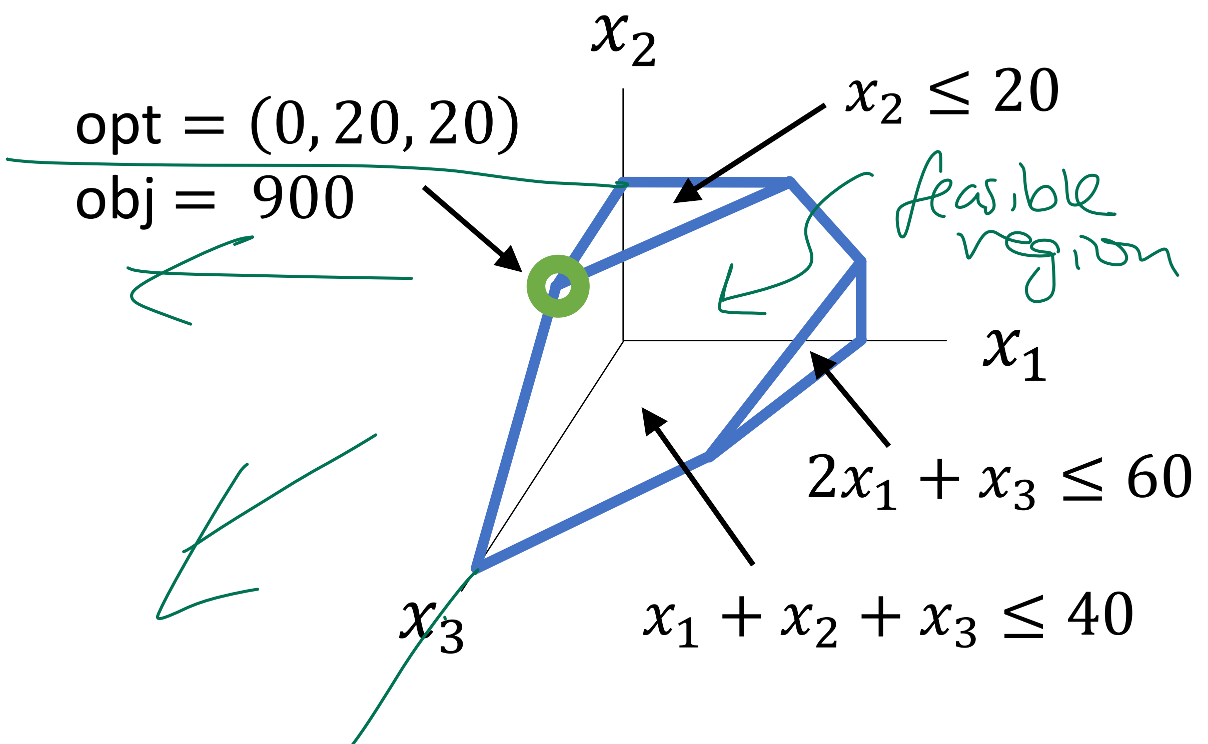

Linear Program (LP)

$x_1 = $ # of Rippers sold
$x_2 = $ # of Ripper Carbons

Objective: $\max 10x_1 + 30x_2$
Subject to: $x_1 \leq 30$
$x_2 \leq 20$
$x_1 + x_2 \leq 40$
$x_1, x_2 \geq 0$

Variables:

- Real numbers = solvable in polynomial time (called LP).
- Integers = not (if P≠NP) solvable in polynomial time (called integer linear program – ILP).

Objective:

- Can be minimization or maximization.
- Must be linear combinations of variables x_i (e.g. $a_1x_1 + \cdots + a_nx_n$ for constants a_i, not $a_ix_1x_2$).

Constraints:

- Can be $\leq$, $\geq$, $=$.
- Must be linear combinations of variables.

A district has an urban area (100,000 voters), suburban area (200,000 voters), and rural area (50,000 voters). A politician decided she needs at least half of the voters in each area to support her. Her campaign has four issues which are popular/unpopular with specific areas. The campaign has estimated the number of voters gained or lost based on each $1 spent advertising an issue. The campaign aims to minimize advertising expenses.

Issue	**Urban**	**Suburban**	**Rural**
Infrastructure	-2	+5	+3
Gun Control	+8	+2	-5
Farm Subsidies	+0	+0	+10
Gasoline Tax	+10	+0	-2