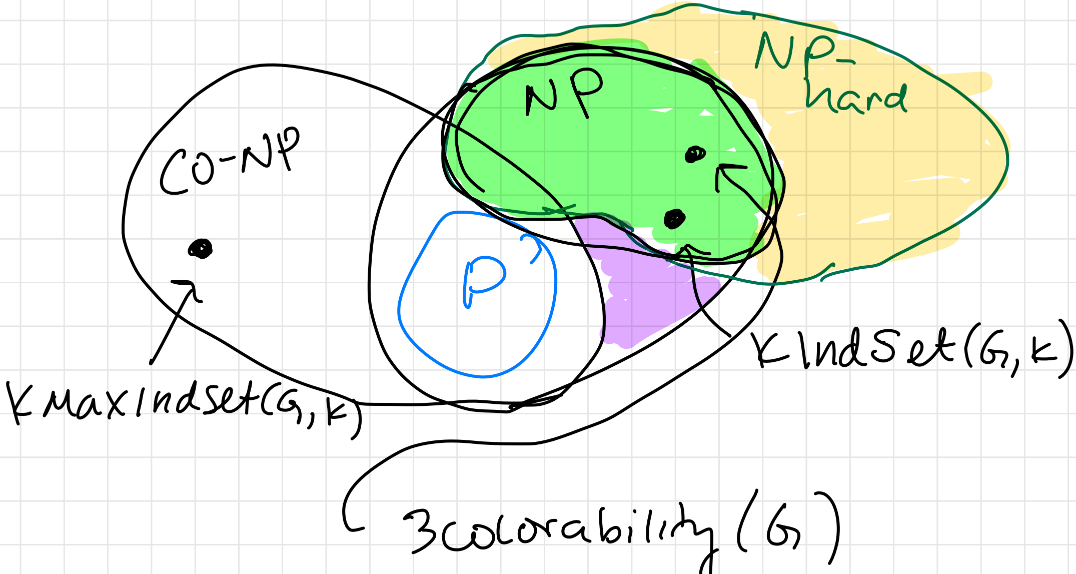



verify yes: poly

NP-complete:
both NP-hard and NP

Claim: Max Independent Set is NP-hard.

Proof: $\boxed{\text{Reduce}}$ known NP-hard problem to Max IS.

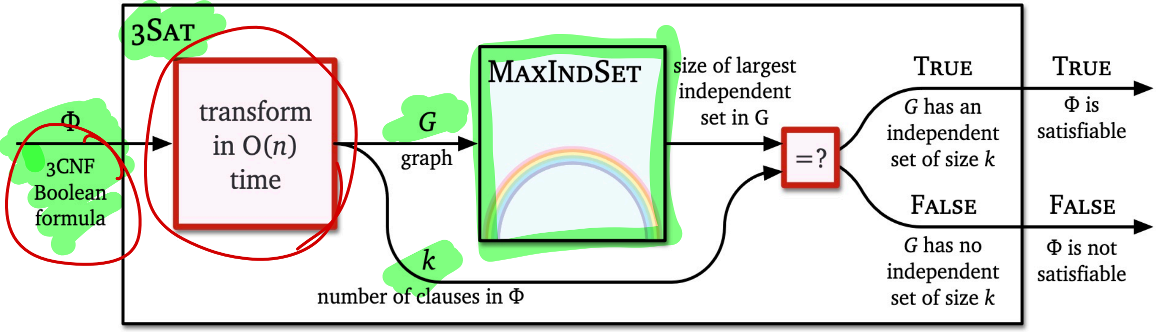
3SAT

literal ↓ literal ↓

$$(a \vee b \vee c) \wedge (\bar{a} \vee b \vee \bar{c})$$

2 clauses
3 literals

* reduction must be polynomial time!




```
Solve3SAT(Φ)
    G = Transform(Φ)
    k = # of clauses in Φ
    n = MaxIndSet(G)
    if n = k:
        return T
    else:
        return F
```

polynomial time

Transform(Φ):

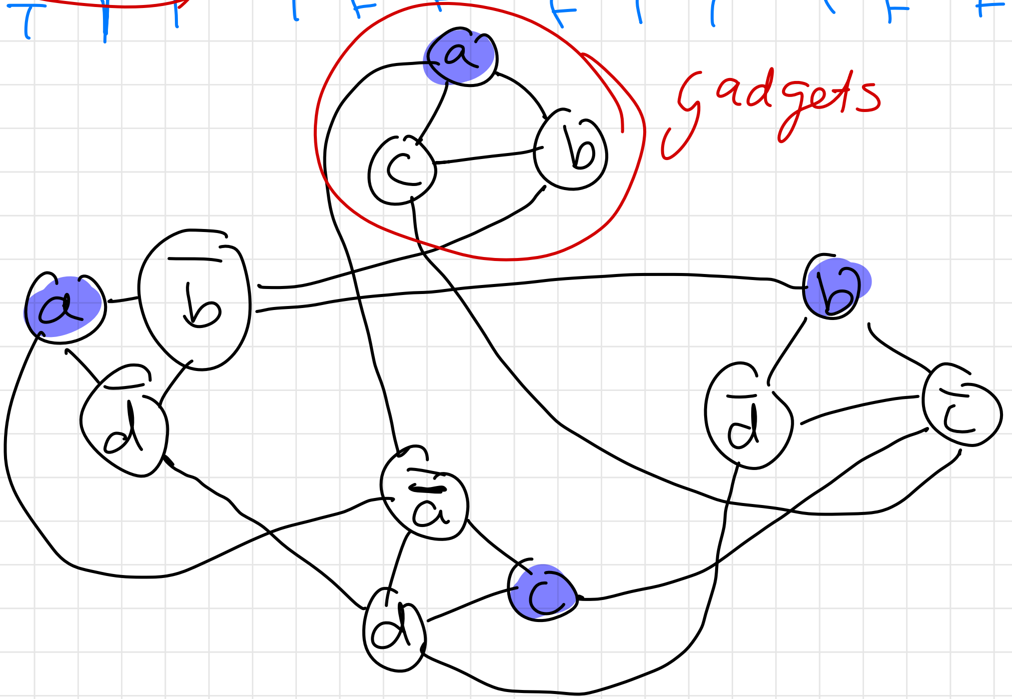
for every clause, create a fully connected group of 3 nodes corresponding to literals in clause.

connect $x$ and $\bar{x}$ for every literal $x$

$$\overset{1}{(a \vee b \vee c)} \wedge \overset{2}{(b \vee \bar{c} \vee \bar{d})} \wedge \overset{3}{(\bar{a} \vee c \vee d)} \wedge \overset{4}{(a \vee \bar{b} \vee \bar{d})}$$

T T T    T F F    F T T    T F F

G:




WTS: $\Phi$ is satisfiable (with k clauses) $\Leftrightarrow$ G has a max IS of size k

$\Rightarrow$ Prove if $\Phi$ satisfiable, then G has max IS of size k.

Assume $\Phi$ has a satisfying assignment.

Fix such a satisfying assignment.

Select one T literal from each clause.

The corresponding nodes form a max independent set in G.

- can't be connected by cross-clause edges
- max size of an IS in G is k since each clause is fully connected.

The max independent set is of size k.

$\Leftarrow$ Assume G has an IS of size k.

Fix such an IS.

The nodes of the IS correspond to a valid truth assignment of $\Phi$, since no $x$ and $\overline{x}$ are selected.

Every clause has a T literal, since no two nodes in a triangle can be selected in IS.

So the set of nodes form a satisfying assn. for $\Phi$.