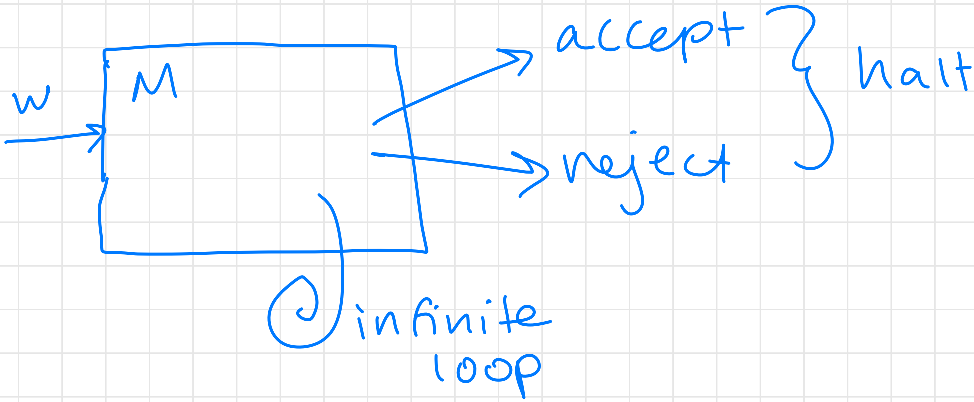
Given an input TM M:



M defines 4 languages:

- $\text{ACCEPT}(M) = \{w \in \Sigma^* : M \text{ accepts } w\}$
- $\text{REJECT}(M) = \{w \in \Sigma^* : M \text{ rejects } w\}$
- $\text{HALT}(M) = \{w \in \Sigma^* : M \text{ halts on } w\}$
  $= \text{ACCEPT}(M) \cup \text{REJECT}(M)$
- $\text{DIVERGE}(M) = \Sigma^* \setminus \text{HALT}(M)$

Let $\langle\ \rangle$ be some encoding scheme for TMs

$\langle M \rangle \in \Sigma^*$ is an encoding of M

$Q = \{q_1, q_2, \dots\}$

δ

$\# q_1 \# q_2 \# \dots \# f_n \# \; \delta(q, a) =$

Let SELFREJECT = $\{\langle M\rangle : M \text{ rejects } \langle M\rangle\}$

can you describe a program that accepts itself?

rejects itself?

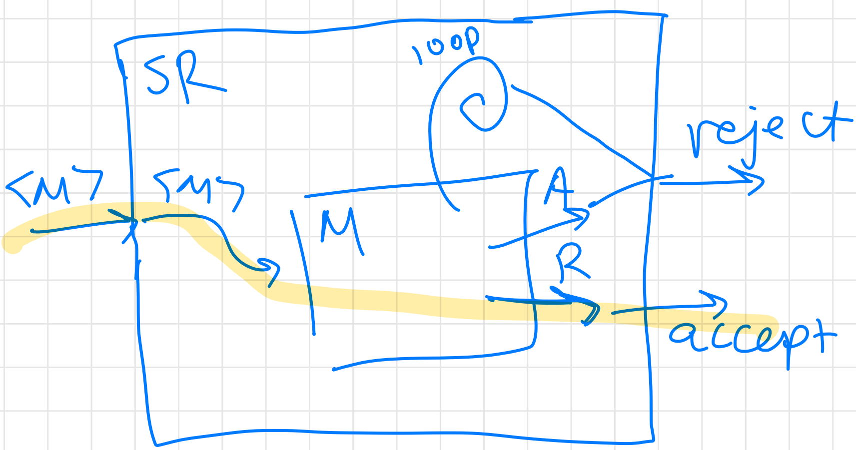
```
input = whatever I get
if "i" in input:
    return True
else:
    return False
```

← SELFACCEPT

Theorem: SELFREJECT is undecidable.

Proof: Suppose not.

Let SR be the TM that decides SELFREJECT.



ACCEPT(SR) = SELFREJECT

DIVERGE(SR) = $\emptyset$

SR accepts <M> iff M rejects <M>

SR accepts <SR> iff SR rejects <SR>

p iff $\neg p$

Contradiction

So SR cannot exist.

HALT = {<M, w> : M halts on w}

Let SELFHALT = {<M> : M halts on <M>}

Theorem: SELFHALT is undecidable.

Proof: Suppose not.

Let SH be a decider for SELFHALT. failed attempt

SH accepts <M> iff M halts on <M>

SH accepts <SH> iff SH halts on <SH>

Let SH^x be a TM built from SH where every transition to an accept state is redirected to a hang state. or reject

SH^x does not halt on $\langle M \rangle$ iff M halts on $\langle M \rangle$

SH^x does not halt on $\langle SH^x \rangle$ iff SH^x halts on $\langle SH^x \rangle$

Contradiction

Theorem: HALT is undecidable.

Proof: Suppose HALT is decidable.

Let H be a TM that decides HALT.

That is, <u>H accepts</u> $\langle M, w \rangle$ iff M halts on w.

So we can decide SELFHALT by running H on $\langle M, \langle M \rangle \rangle$

input w.

But SELFHALT is undecidable.

So H can't exist.