CSCI 432/532, Spring 2024
Problem Session 9

---

1. Let M be an arbitrary Turing machine. Prove the following claims.

   (a) There is a Turing machine M^R such that

   $$\textsc{Accept}(M^R) = \textsc{Reject}(M)$$

   and

   $$\textsc{Reject}(M^R) = \textsc{Accept}(M).$$

   (b) There is a Turing machine M^A such that

   $$\textsc{Accept}(M^A) = \textsc{Accept}(M)$$

   and

   $$\textsc{Reject}(M^A) = \emptyset.$$

   (c) There is a Turing machine M^H such that

   $$\textsc{Accept}(M^H) = \textsc{Halt}(M)$$

   and

   $$\textsc{Reject}(M^H) = \emptyset.$$

2. Let $\textsc{SelfAccept} = \{\langle M \rangle : M \text{ accepts } \langle M \rangle\}$. Prove that $\textsc{SelfAccept}$ is undecidable.

3. Let $\textsc{Accept} = \{\langle M, w \rangle : M \text{ accepts } w\}$. Prove that $\textsc{Accept}$ is undecidable.

To think about later:

1. Prove that each of the following languages is undecidable.

   (a) $\textsc{AlwayAccept} = \{\langle M \rangle : \textsc{Accept}(M) = \Sigma^*\}$

   (b) $\textsc{AlwayReject} = \{\langle M \rangle : \textsc{Reject}(M) = \Sigma^*\}$

   (c) $\textsc{AlwayHalt} = \{\langle M \rangle : \textsc{Halt}(M) = \Sigma^*\}$

   (d) $\textsc{AlwayDiverge} = \{\langle M \rangle : \textsc{Diverge}(M) = \Sigma^*\}$