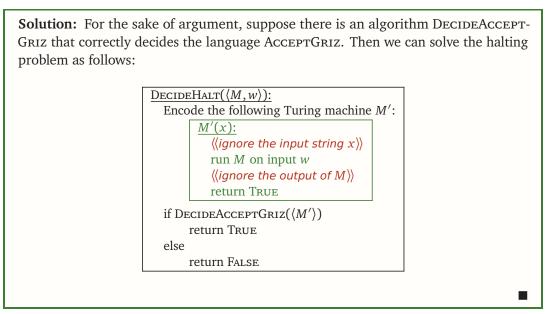# Problem Session 10

1. Below is the start of a proof that the language $\textsc{AcceptGriz} := \{\langle M\rangle \mid M \text{ accepts the string } \texttt{GRIZ}\}$ is undecidable.

> **Solution:** For the sake of argument, suppose there is an algorithm $\textsc{DecideAccept-Griz}$ that correctly decides the language $\textsc{AcceptGriz}$. Then we can solve the halting problem as follows:
>
> > $\underline{\textsc{DecideHalt}(\langle M, w\rangle)}$:
> > Encode the following Turing machine M':
> > > $\underline{M'(x)}$:
> > > $\langle\!\langle \text{ignore the input string } x \rangle\!\rangle$
> > > run M on input w
> > > $\langle\!\langle \text{ignore the output of } M \rangle\!\rangle$
> > > return $\textsc{True}$
> >
> > if $\textsc{DecideAcceptGriz}(\langle M'\rangle)$
> > > return $\textsc{True}$
> > else
> > > return $\textsc{False}$
>
> ∎

   (a) List the four distinct Turing machines that are at play in this reduction and describe their role.

   (b) Prove that this reduction is correct by proving both

   - if M halts on input w, then $\textsc{DecideHalt}$ accepts on $(\langle M, w\rangle)$, and
   - if $\textsc{DecideHalt}$ accepts on $(\langle M, w\rangle)$, then M halts on input w.

2. Prove that each of the following languages is undecidable.

   (a) $\textsc{AlwaysReject} = \{\langle M\rangle : \textsc{Reject}(M) = \Sigma^*\}$

   (b) $\textsc{AlwaysHalt} = \{\langle M\rangle : \textsc{Halt}(M) = \Sigma^*\}$

   (c) $\textsc{AlwaysDiverge} = \{\langle M\rangle : \textsc{Diverge}(M) = \Sigma^*\}$