CSCI 432/532, Spring 2025 Problem Session 15

1. The *edge-disjoint path problem* takes in a directed graph *G* with special nodes *s* and *t* and asks for the largest number of paths from *s* to *t* that do not share any edges. (They may share vertices, however!)

Describe a reduction from the edge-disjoint path problem to the maximum flow problem.

2. The *maximum matching problem* takes in a (undirected) bipartite graph *G* and asks for the largest collection of edges from *G* such that no two edges share an endpoint.

Describe a reduction from the maximum matching problem to the maximum flow problem.

3. We can imagine a version of the maximum flow problem where vertices (except *s* and *t*) also have capacities, and we require that the total flow into (or, equivalently, out of) a vertex v to be less than or equal to that capacity. (Note that we still have edge capacities in this version.) We call this the *vertex-capacitated maximum flow problem*.

Give a reduction from the vertex-capacitated maximum flow problem to the maximum flow problem.